Unit Title: Reproducing Bacterial Rabbits

INSTRUCTIONAL UNIT AUTHORS

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BASED ON A CURRICULUM OVERVIEW SAMPLE AUTHORED BY

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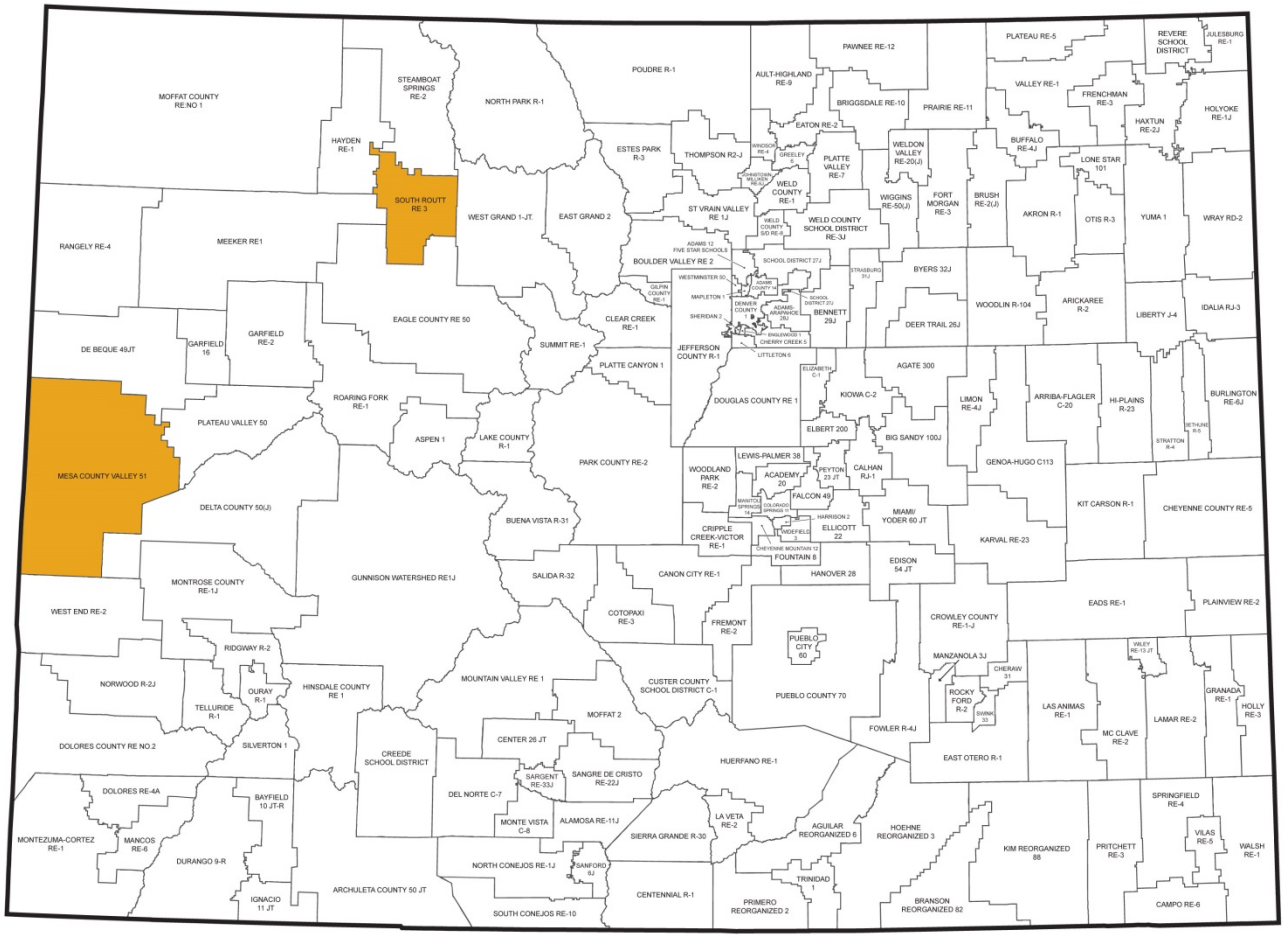

This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.

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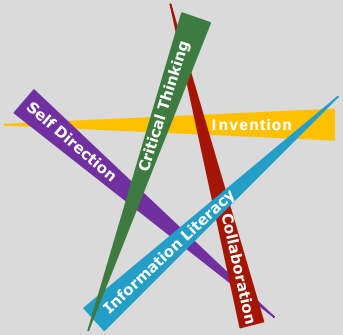
Colorado Teacher-Authored Sample Instructional Unit

Content Area	Mathematics	Grade Level	High School
Course Name/Course Code	Integrated Math 2		

Standard	Grade Level Expectations (GLE)	GLE Code
1. Number Sense, Properties, and Operations	1. The complex number system includes real numbers and imaginary numbers	MA10-GR.HS-S.1-GLE.1
	2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations	MA10-GR.HS-S.1-GLE.2
2. Patterns, Functions, and Algebraic Structures	1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables	MA10-GR.HS-S.2-GLE.1
	2. Quantitative relationships in the real world can be modeled and solved using functions	MA10-GR.HS-S.2-GLE.2
	3. Expressions can be represented in multiple, equivalent forms	MA10-GR.HS-S.2-GLE.3
	4. Solutions to equations, inequalities and systems of equations are found using a variety of tools	MA10-GR.HS-S.2-GLE.4
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics condense the information in data sets into usable knowledge	MA10-GR.HS-S.3-GLE.1
	2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions	MA10-GR.HS-S.3-GLE.2
	3. Probability models outcomes for situations in which there is inherent randomness	MA10-GR.HS-S.3-GLE.3
4. Shape, Dimension, and Geometric Relationships	1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically	MA10-GR.HS-S.4-GLE.1
	2. Concepts of similarity are foundational to geometry and its applications	MA10-GR.HS-S.4-GLE.2
	3. Objects in the plane can be described and analyzed algebraically	MA10-GR.HS-S.4-GLE.3
	4. Attributes of two- and three-dimensional objects are measurable and can be quantified	MA10-GR.HS-S.4-GLE.4
	5. Objects in the real world can be modeled using geometric concepts	MA10-GR.HS-S.4-GLE.5

Colorado 21st Century Skills



Critical Thinking and Reasoning: *Thinking Deeply, Thinking Differently*

Information Literacy: *Untangling the Web*

Collaboration: *Working Together, Learning Together*

Self-Direction: *Own Your Learning*

Invention: *Creating Solutions*

Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Titles	Length of Unit/Contact Hours	Unit Number/Sequence
Reproducing Bacterial Rabbits	7 weeks	1

Unit Title	Reproducing Bacterial Rabbits		**Length of Unit**	6 weeks
Focusing Lens(es)	Modeling Relationship	**Standards and Grade Level Expectations Addressed in this Unit**	MA10-GR.HS-S.1-GLE.2 MA10-GR.HS-S.2-GLE.2 MA10-GR.HS-S.2-GLE.3 MA10-GR.HS-S.2-GLE.4	
Inquiry Questions (Engaging-Debatable):	• What is the best way of paying off debt on multiple credit cards? • What financial phenomena can be modeled with exponential and linear functions? (MA10-GR.HS-S.2-GLE.2-IQ.3)			
Unit Strands	Number and Quantity: The Real Number System Algebra: Seeing Structure in Expressions Algebra: Creating Equations Functions: Building Functions Functions: Interpreting Functions			
Concepts	Sums, products, rational numbers, logarithms, inverse, exponential, functions, integer exponents, rational exponents, properties, transformations, expressions, average rate of change, classes of functions, translations, graph			

Generalizations **My students will Understand that…**	**Guiding Questions** **Factual**	**Conceptual**
The sums and products of rational numbers remain in the set of rational numbers. (MA10-GR.HS-S.1-GLE.1-EO.b)	What is product or sum of two irrational numbers?	Why is the sum or product of two rational numbers always rational? Why are the sum and products of irrational numbers with rational numbers always irrational?
The properties of integer exponents extend to rational exponents. (MA10-GR.HS-S.1-GLE.1-EO.a)	What are the properties of exponents? What is the relationship between rational exponents and radicals? How can properties of exponents be used to transform rational expressions into radical expressions or vice versa? How are radical expressions simplified?	Why do we need both radicals and rational exponents?
Properties of exponents and operations can transform expressions to facilitate interpretation of the quantities represented by the expression. (MA10-GR.HS-S.2-GLE.1-EO.c.) and (MA10-GR.HS-S.2-GLE.3-EO.a.ii, b.i.3)	What is the impact on the graph of transforming an expression?	Why do linear and exponential functions model so many situations? Why is the domain of a sequence a subset of the integers?

The interpretation of the parameters of equations and inequalities must consider real world contexts. (MA10-GR.HS-S.2-GLE.2-EO.b.i)	What is a coefficient? How do you choose coefficients given a set of data?	Why might it be necessary to transform an exponential expression to better interpret the context of situation?
Mathematicians compare average rates of change over a specified interval to determine the increase or decrease of a function relative to another function. (MA10-GR.HS-S.2-GLE.1-EO.b.iii)	How are the starting population and the growth factor represented in an exponential function? How do you calculate average rate of change of an exponential function? How does the average rate of change impact the behavior of a function over the entire span of the function?	How is the average rate of change represented in the graph and table of an exponential function?
Exponential functions provide the means to model constant rates of growth.	What is a constant rate of growth? How can you determine a constant rate of growth from a graph, table, and equation?	Why are linear functions modeled with constant differences and exponential functions with constant rates of growth?

Key Knowledge and Skills: **My students will…**	*What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.*

- Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
- Rewrite expressions involving radicals and rational exponents using the properties of exponents. (MA10-GR.HS-S.1-GLE.1-EO.a.ii)
- Use the structure of an expression to identify ways to rewrite it. (MA10-GR.HS-S.2-GLE.3-EO.a.ii)
- Interpret key features of graphs and table, for an exponential function, in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (MA10-GR.HS-S.2-GLE.1-EO.b.i)
- Calculate and interpret the average rate of change of an exponential function (presented symbolically or as a table) over a specified interval and estimate the rate of change from a graph. (MA10-GR.HS-S.2-GLE.1-EO.b.iii)
- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (MA10-GR.HS-S.2-GLE.1-EO.c.iii)
- Graph exponential functions, showing intercepts and end behavior. (MA10-GR.HS-S.2-GLE.1-EO.c.iv)
- Use the properties of exponents to interpret expressions for exponential functions. (MA10-GR.HS-S.2-GLE.3-EO.b.i.3)
- Compare properties of two exponential functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (MA10-GR.HS-S.2-GLE.1-EO.c.v.3)
- Use the properties of exponents to transform expressions for exponential functions. (MA10-GR.HS-S.2-GLE.1-EO.c.v.2)
- Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. (MA10-GR.HS-S.1-GLE.1-EO.b)
- Determine an explicit expression, a recursive process, or steps for calculation from an exponential context. (MA10-GR.HS-S.2-GLE.1-EO.d.i.1)
- Create exponential equations and inequalities in one variable and use them to solve problems. (MA10-GR.HS-S.2-GLE.4-EO.a.i)
- Analyze the impact of interest rates on a personal financial plan. (MA10-GR.HS-S.2-GLE.2-EO.d.i) *
- Evaluate the costs and benefits of credit. (MA10-GR.HS-S.2-GLE.2-EO.d.ii) *
- Analyze various lending sources, service and financial institutions. (MA10-GR.HS-S.2-GLE.2-EO.d.iii) *

*** Denotes connection to Personal Financial Literacy (PFL)**

Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline. EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *"Mark Twain exposes the hypocrisy of slavery through the use of satire."*	
A student in _____ can demonstrate the ability to apply and comprehend critical language through the following statement(s):	*I know how use to properties of exponents to create an exponential equation to model constant rates of growth.*
Academic Vocabulary:	Rewrite, structure, identify, key features, graphs, tables, descriptions, relationships, calculate, interpret, compare, graphically, numerically, verbal descriptions, combine
Technical Vocabulary:	Sums, products, rational numbers, logarithms, inverse, exponential, functions, integer exponents, rational exponents, properties, transformations, expressions, average rate of change, classes of functions, translations, radicals, rational, square root, cube root, piece-wise-defined functions, step functions, absolute value functions, explicit expression, recursive process, linear, quadratic

Unit Description:	This unit focuses on exponential functions embedded in our everyday financial lives. Students explore credit cards, loans and savings accounts. As they explore these topics, they are motivated to solve exponential equations and become fluent in interpreting exponential expressions by using the properties of exponents to rewrite expressions. Students are introduced to the concept of the mathematical constant e, through the context of continuously compounding interest. Students also explore the attributes of classes of functions including their inverses, which introduce the concept of a logarithm. Students will further explore classes of functions and the impact of multiplying or adding a constant in the next unit and the concept of logarithm is developed further in Integrated III.
Unit Generalizations	
Key Generalization:	Properties of exponents and operations can transform expressions can functions to facilitate interpretation of the quantities represented by the expression.
Supporting Generalizations:	The sums and products of rational numbers remain in the set of rational numbers.
	The properties of integer exponents extend to rational exponents.
	Mathematicians compare average rates of change over a specified interval to determine the increase or decrease of a function relative to another function.
	Exponential functions provide the means to model constant rates of growth.

Performance Assessment: *The capstone/summative assessment for this unit.*	
Claims: (Key generalization(s) to be mastered and demonstrated through the capstone assessment.)	Properties of exponents and operation can transform expressions to facilitate interpretation of the quantities represented by the expression.
Stimulus Material: (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)	You would like to purchase a vehicle. Your parents might be willing to co-sign for a loan but you need to present to them a plan that is well-researched about the type of vehicle you want to purchase, the cost of financing (loan) including sales tax on the purchase price, depreciation of the vehicle, and additional monthly and yearly expenses such as registration, fuel, repairs, and insurance. Specifically, you will need to research possible loans and use the equation below to determine monthly payments: $$A = P\frac{r(1+r)^n}{(1+r)^n - 1}$$ where *A* is the payment Amount per month, *P* is the initial principal (loan amount), *r* is interest rate per month (note this is rate per month not year), *n* is the total number of payments or months Create an equation to determine the value of your vehicle each month after it was purchased using the average depreciations below:

	<table><tr><td>Age of Vehicle in Years</td><td>Average Yearly Depreciation</td></tr><tr><td>0 to 4</td><td>15%</td></tr><tr><td>5 to 9</td><td>4%</td></tr><tr><td>10 and older</td><td>1%</td></tr></table> Visually display the relationship between your depreciated value and your accumulated monthly loan payments and explain the meaning of this relationship in relation to your decision to buy the vehicle. Determine the total estimated cost of owning your vehicle over the life of the loan including the additional expenses of registration, insurance, fuel, maintenance and repair and calculate how many hours you would need to work a month to pay for your vehicle. Finally, compare this plan to waiting a year during which you save the cost of the car each month in a 2% saving account and was used for down payment on the car.
Product/Evidence: (Expected product from students)	Students will produce a plan for purchasing a car that includes the following: A description of the car(s) (make, model, year, mileage) and an explanation for their choices. Amount of sales tax, registration, insurance, fuel costs, maintenance and repair costs Calculation of monthly payments using the equation below for at least two different loan options $A = P\frac{r(1+r)^n}{(1-r)^n - 1}$ where *A* is the payment Amount per month, *P* is the initial principal (loan amount), *r* is interest rate per month (note this is rate per month not year), *n* is the total number of payments or months. Calculate *monthly* depreciation costs: http://www.free-online-calculator-use.com/car-depreciation-calculator.html (depreciation calculator) and find an equation that will determine the value of the car each month after it was purchased. Graph the value of the vehicle over the life of the loan compared to the amount of money spent purchasing the vehicle for various loans and explain the meaning of the intersection of the functions. Determine the total cost for owning the car for each loan type during the life of the loan including the additional expenses of (registration, insurance, fuel, maintenance and repair) and the average monthly cost. Determine how many hours a month at a minimum wage job is required to pay for the cost of the car. Determine how much money would be saved if the amount of money required to own a car was saved for one year in a 2% annual interest rate savings account and was used as a down payment for the car. Make a recommendation about how, when or if they should purchase a car.
Differentiation: (Multiple modes for student expression)	http://www.free-online-calculator-use.com/car-buying-calculator.html (online calculator that explains expenses associated with owning a vehicle and calculates the total and monthly cost; and, compares buying two cars by looking at financing, depreciation, fuel costs, insurance, etc.) Students can explain why this equation and the equation in the stimulus are equivalent: $P = A\frac{1-(1+r)^{-n}}{r}$ Students can explore leasing options and compare them to purchasing a new or used car.

Texts for independent reading or for class read aloud to support the content

Informational/Non-Fiction	Fiction
The Young Investor by Katherine R Bateman (Lexile Level currently not available) *Straight Talk About Money* by Rendon, Marion B & Kranz, Rachel (Lexile Level currently not available) *Real World Math: Money & Other Numbers in Your Life* by Donna Guthrie (Lexile Level currently not available)	*Make Lemonade* by Virginia Wolff (Lexile Level 890)

Ongoing Discipline-Specific Learning Experiences

1.	Description:	Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others	Teacher Resources:	http://www.insidemathematics.org/index.php/standard-3 (examples of constructing viable arguments) http://quizlet.com/22134361/cpm-index-cards-of-teaching-strategies-flash-cards/ (teaching strategies to encourage class discussions)
			Student Resources:	N/A
	Skills:	Provide justification for arguments through a series of logical steps while using correct mathematical vocabulary. Analyze and critique the arguments of other students	Assessment:	Students justify their reasoning about exponential functions and classifications of numbers. Students use precise language such as exponential, rate of growth, common difference, rational, and irrational. Students can also critique the reasoning of others.
2.	Description:	Think/work like a mathematician – Engaging in the practice of modeling the solution to real world problems	Teacher Resources:	http://www.corestandards.org/Math/Content/HSM (Common Core State Standards description of the modeling process) http://blog.mrmeyer.com/?p=16301 (Dan Meyer discussion on modeling) http://threeacts.mrmeyer.com (Examples of 3-act problems)
			Student Resources:	N/A
	Skills:	Model real world problems mapping relationships with appropriate models of functions, analyze relationships to draw conclusions, interpret results in relation to context, justify and defend the model, and reflect on whether results make sense	Assessment:	Modeling Problems Students use exponential and linear functions to model real world contexts with an emphasis on financial contexts. Students will be able to draw conclusions and interpret their models in relation to the context to determine if their model makes sense.

3.	Description:	Mathematicians are fluent with graphing functions and interpreting key features of the graphs.	Teacher Resources:	https://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec1_mod5_features_tn_71713.pdf (a module focused on graphing functions and interpreting key features of graphs, p. 32-33 are particularly helpful)
			Student Resources:	http://www.thatquiz.org/tq-0/math/algebra/ (linear equation practice)
	Skills:	Graphing key features and recognizing relationships between data sets and classes of functions.	Assessment:	Fluency Problems Students build fluency with geometric transformations with consistent practice visualizing the result of rotations, reflections and translations.

Prior Knowledge and Experiences

Student familiarity with exponential functions and properties of exponents will support their work throughout this unit. Students will also encounter a new irrational number, e. Students might benefit from a reminder about the difference between rational and irrational numbers.

Learning Experience # 1

The teacher may provide a variety of different representations of equivalent exponential equations so that students can graph functions and explain why equations are equivalent using both symbolical and graphical representations (ex.: y = 5 and y = $\left(5^{\frac{1}{3}}\right)^3$)

Iconic: Students can use a graphing calculator to represent the equivalence of different forms of exponential equations.
Symbolic: Students can symbolically manipulate exponential expressions to show equivalence.

Teacher Notes:	Students may need a refresher on the equivalence of equations such as $\sqrt{2} = 2^{\frac{1}{2}}$ prior to the start of this learning experience.
Generalization Connection(s):	The properties of integer exponents extend to rational exponents.
Teacher Resources:	http://education.ti.com/en/us/activity/detail?id=E52BBFFACB2C4FB9853BA9F367ED135A&ref=/en/us/activity/search/subject?d=6B854F0B5CB6499F8207E81D1F3A25E6&s=B843CE852FC5447C8DD88F6D1020EC61&sa=2D7AB06424004125A392EB9A075CABC0&t=5BF5C8FD21C94240A86443E5C4F73B11 (graphing calculator activity discovering the rules of exponents) https://www.illustrativemathematics.org/content-standards/HSN/RN/A/1/tasks/1866 (examples of tasks focused on equivalent exponential expressions) https://www.illustrativemathematics.org/content-standards/HSN/RN/A/1/tasks/385 (example of a modeling task focused on equivalent equations) https://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec2_mod3_quadeq_tn_83113.pdf (Math Vision Project 3.1 – 3.4 focused on equivalent exponential equations) https://www.engageny.org/resource/algebra-ii-module-3 (Engage NY lesson 3 – 5, focused on equivalent exponential equations)

<table>
<tr><td>Student Resources:</td><td colspan="2">http://education.ti.com/en/us/activity/detail?id=E52BBFFACB2C4FB9853BA9F367ED135A&ref=/en/us/activity/search/subject?d=6B854F0B5CB6499F8207E81D1F3A25E6&s=B843CE852FC5447C8DD88F6D1020EC61&sa=2D7AB06424004125A392EB9A075CABC0&t=5BF5C8FD21C94240A86443E5C4F73B11 (student investigations using graphing calculator activity discovering the rules of exponents)
https://www.khanacademy.org/math/pre-algebra/exponents-radicals/World-of-exponents/v/exponents-warmup (basic rules of exponents)
http://www.algebralab.org/lessons/lesson.aspx?file=Algebra_ExponentsRules.xml (simplifying exponential expressions)
https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/introduction-to-rational-exponents-and-radicals/v/basic-fractional-exponents (rational exponents and radical expressions)
http://catalog.flatworldknowledge.com/bookhub/128?e=fwk-redden-ch08_s05 (notes and examples of rules of exponents and simplifying exponential expressions)</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this lesson should be able to answer questions such as:
• What are the properties of exponents?
• What is the relationship between rational exponents and radicals?
• How can the properties of exponents be used to make equivalent expressions?
• How can properties of exponents be used to transform rational expressions into radical expressions or vice versa?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>https://wvde.state.wv.us/strategybank/FrayerModel.html (Frayer model template for vocabulary words)</td><td>Students can simplify exponential expressions and transform rational expression into radical expressions using vocabulary support and rules, for example: $a^m \bullet a^n = a^{m+n}$.</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>https://www.illustrativemathematics.org/content-standards/tasks/385 (tasks involving radical expressions)
https://www.khanacademy.org/math/algebra/rational-exponents-and-radicals/rational-exponents-and-the-properties-of-exponents/v/radical-equivalent-to-rational-exponents-2 (resources involving radical expressions)</td><td>Students can rewrite a radical expression with multiple variables as an exponential expression</td></tr>
<tr><td>Key Knowledge and Skills:</td><td colspan="2">• Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
• Rewrite expressions involving radicals and rational exponents using the properties of exponents.</td></tr>
<tr><td>Critical Language:</td><td colspan="2">Rewrite, calculate, compare, graphically, numerically, integer exponents, rational exponents, properties, expressions, radicals, rational, square root, cube root, explicit expression</td></tr>
</table>

Learning Experience # 2
Teacher may provide examples of rational and irrational numbers and their sums and products so that students can classify sums or products as rational or irrational.

Iconic: Students can sort numbers into two lists, rational and irrational and decide if sums and products of combinations of rational and irrational numbers produce a solution that is rational or irrational on an "Always, Sometimes, Never" chart. *Symbolic:* Students can compute sums and products of rational and irrational numbers and classify the result as rational or irrational.		
Teacher Notes:	The Shell Center lesson in the teacher resources provides resources for the always, sometimes, never chart. Students might benefit from a reminder of the concepts of rational and irrational numbers.	
Generalization Connection(s):	The sums and products of rational numbers remain in the set of rational numbers.	
Teacher Resources:	http://map.mathshell.org/materials/download.php?fileid=1267 (Shell Center lesson for sorting into "always, sometimes, never") https://www.khanacademy.org/math/algebra/rational-and-irrational-numbers/irrational-numbers/v/introduction-to-rational-and-irrational-numbers (introduction to rational and irrational numbers as part of the number system) https://www.illustrativemathematics.org/content-standards/HSN/RN/A/2 (tasks focused generalizing operations on rational and irrational numbers)	
Student Resources:	N/A	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: What is product or sum of two irrational numbers? Why is the sum or product of two rational numbers always rational? Why are the sum and products of irrational numbers with rational numbers always irrational?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	https://wvde.state.wv.us/strategybank/FrayerModel.html (Frayer model template for vocabulary words)	Students can classify numbers using vocabulary support.
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	N/A
Key Knowledge and Skills:	• Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	
Critical Language:	Rational numbers, irrational numbers, real numbers, exceptions, sum, product	

Learning Experience # 3
Teacher may provide examples of savings account/car loan interest rates so that students can analyze options between saving versus financing a major purchase. *Enactive*: Students can act out a scenario where one student can act as bank while the other as the customer, then students can make a prediction about the best bank to invest with or borrow from and justify their choice to a partner. *Iconic*: Students can make a table representing how much money they would have with each bank after (e.g., 1 year, 2 years, 3 years…) or how much interest they would have paid

<table>
<tr><td colspan="3">with each loan. Students can analyze their table and revisit their prediction about the best bank.
Symbolic: Students can analyze the table and create an equation using function notation. Students can interpret the parameters in terms of the context and predict how much money will be in the account after 10 years or how much interest they would have paid in 5 years. They also can determine how much money is in their account after .5 years, or 7.5 years.</td></tr>
<tr><td>Teacher Notes:</td><td colspan="2">Students may need a review of how to calculate exponential growth.</td></tr>
<tr><td>Generalization Connection(s):</td><td colspan="2">Properties of exponents and operations to transform expressions can functions to facilitate interpretation of the quantities represented by the expression.</td></tr>
<tr><td>Teacher Resources:</td><td colspan="2">http://www.bankrate.com/calculators/index-of-checking-savings-calculators.aspx (interest rate calculator)
https://www.engageny.org/resource/algebra-ii-module-1 (Engage New York Lesson 13 focuses on differences of squares)</td></tr>
<tr><td>Student Resources:</td><td colspan="2">http://www.webmath.com/amort.html (loan amortization calculator)
http://www.math.com/students/calculators/source/compound.htm (interest calculator)</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this lesson should be able to answer questions such as:
• How can interest rates from various lending sources and financial institutions be used to analyze personal financial plans?
• Why is it important to shop around for the best interest rates (loans or savings account)?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>https://wvde.state.wv.us/strategybank/FrayerModel.html (Frayer model template for vocabulary words)</td><td>Students are able to compare investments and loans using their vocabulary sheet.</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>https://bigfuture.collegeboard.org/pay-for-college/loans/student-loan-calculator (student loan calculator)</td><td>Students are able to choose a college and create a financial plan for attending college.</td></tr>
<tr><td>Key Knowledge and Skills:</td><td colspan="2">• Use the properties of exponents to interpret expressions for exponential functions.
• Use the structure of an expression to identify ways to rewrite it
• Use the properties of exponents to transform expressions for exponential functions.
• Compare properties of two exponential functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
• Determine an explicit expression, a recursive process, or steps for calculation from an exponential context.
• Analyze the impact of interest rates on a personal financial plan.
• Evaluate the costs and benefits of credit.
• Analyze various lending sources, service and financial institutions. (MA10-GR.HS-S.2-GLE.2-EO.d.iii) *</td></tr>
<tr><td>Critical Language:</td><td colspan="2">Rewrite, structure, graphs, tables, descriptions, relationships, calculate, interpret, compare, graphically, numerically, verbal descriptions, combine, rational numbers, exponential, functions, integer exponents, rational exponents, properties, expressions, rational, explicit expression</td></tr>
</table>

Learning Experience # 4

The teacher may provide students with scenarios where interest on a savings account is compounded more and more often (yearly, monthly, daily, hourly, every minute, every second, etc.) so that students can explore the limit of compounding at a particular interest rate.

Iconic: Students can explore graphically and in tables how compounding more often leads to more money, but that the increase in interest income soon becomes something that makes almost no difference. This is in essence a limit—the limit on how much interest income one can earn at a specific interest rate, no matter how often the account is compounded.

Symbolic: The students can explore what happens when an account is compounded an infinite number of times (compounded continuously) and derive the formula for compounding continuously ($y = Pe^{rt}$) as shown below in the teacher notes.

Teacher Notes:	Much of the derivation of the number e given above uses the idea of a limit, and the teacher may choose to use this terminology or not during this exploration. The teacher may show students how to manipulate the expression, $y = P\left(1+\frac{r}{n}\right)^{nt}$, where n is the number of times the account is compounded each year, P is the principle, r is the interest rate, and t is the number of years the account is compounded. By first rewriting the expression as $y = P\left(1+\frac{1}{\frac{n}{r}}\right)^{\frac{n}{r}t}$ the can explore the limit (horizontal asymptote) of $y = \left(1+\frac{1}{\frac{n}{r}}\right)^{\frac{n}{r}}$ as n goes to infinity. This will be e, which is why the limit of the expression $y = P\left(1+\frac{r}{n}\right)^{nt}$ is $y = Pe^{rt}$ This is the first time that students have been introduced to the number e. The teacher may want to point out that e is analogous to pi, an irrational number that is discovered, not invented. With the introduction of e, the teacher may introduce the natural logarithm as well but this is not necessary until Integrated III.
Generalization Connection(s):	Properties of exponents and operations can transform expressions for exponential functions to facilitate interpretation of the quantities represented by the expression.
Teacher Resources:	http://betterexplained.com/articles/demystifying-the-natural-logarithm-ln/ (explanation about the natural logarithm) https://www.khanacademy.org/math/algebra2/exponential_and_logarithmic_func/continuous_compounding/v/introduction-to-compound-interest-and-e (Kahn video on natural logarithm) https://www.youtube.com/watch?v=yTfHn9Aj7UM (video about the natural logarithm)
Student Resources:	https://www.desmos.com/calculator (graphing applet)

Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: • What is the number *e*? • What is the natural logarithm? (extension) • Why it is necessary to use *e* to compound interest continuously? What are examples of modeling with *e (*other than compound interest)?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	https://learnzillion.com/lessons/220-understand-and-apply-the-definition-of-irrational-numbers (reminder of the concept of irrational numbers as they learn that *e* is irrational)	Students can explain why e, just like π is a quantity/number and not a variable.
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	https://www.youtube.com/watch?v=TINfzxSnnIE (video that explores 0.999999…..=1)	Students can explain the similarity between the concept of e and the concept that 0.999999…..=1.
Key Knowledge and Skills:	• Use the properties of exponents to transform expressions for exponential functions (with both rational and real exponents).	
Critical Language:	e , irrational, limit, horizontal asymptote, compound continuously	

Learning Experience # 5	
Teacher may provide examples of models of exponential growth and decay so that students can recognize different forms of exponential models. *Enactive*: Students can use the provided models to calculate amounts after a specified amount of time. *Iconic*: Students can make a table or a graph representing amounts during a specified amount of time. *Symbolic*: Students can create algebraic models (equations and inequalities) and interpret key features of graphs and tables based on their model. Students can recognize different but equivalent forms of exponential models.	
Teacher Notes:	Key features of a graph students can explore include average rates of change, intercepts, intervals of increasing/decreasing/positive/negative, relative maximum/minimum, symmetries, and end behavior.
Generalization Connection(s):	Exponential functions provide the means to model constant rates of growth. Mathematicians compare average rates of change over a specified interval to determine the increase or decrease of a function relative to another function.
Teacher Resources:	http://www.mathsisfun.com/algebra/exponential-growth.html (sample word problems for exponential growth) https://www.engageny.org/resource/algebra-i-module-3 (lessons 5 - 7 focus on exponential growth/decay)
Student Resources:	https://www.khanacademy.org/math/algebra2/exponential_and_logarithmic_func/exponential-modeling/v/word-problem-solving-exponential-growth-and-decay (exponential growth and decay word problems)

Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: • How is exponential modeling used to describe real world situations? • How the key features of graphs can be used to analyze exponential models? • How is the average rate of change represented in the graph and table of an exponential function?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	https://wvde.state.wv.us/strategybank/FrayerModel.html (Frayer model template for vocabulary words)	Students can model exponential growth and decay using their vocabulary.
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	https://learnzillion.com/lesson_plans/437-recognize-the-differences-between-exponential-and-linear-growth-by-examining-salary-increases (comparing exponential and linear growth with salary increases)	Students can create a presentation about whether salary growth should be modeled with exponential or linear growth and the implications for choices of professions.
Key Knowledge and Skills:	• Interpret key features of graphs and table, for an exponential function, in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. • Calculate and interpret the average rate of change of an exponential function (presented symbolically or as a table) over a specified interval and estimate the rate of change from a graph • Create exponential equations and inequalities in one variable and use them to solve problems.	
Critical Language:	Rewrite, structure, identify, key features, graphs, tables, descriptions, relationships, calculate, interpret, compare, graphically, numerically, verbal descriptions, rational numbers, logarithms, exponential, functions, integer exponents, rational exponents, properties, expressions, average rate of change, rational, explicit expression, recursive process	

Learning Experience # 6	
Teacher may provide examples of graphs and equations of families of functions (exponential, square root, cube root, and piecewise-defined including step and absolute value) so that students can compare average rates of change between two non-linear functions on the same interval. *Enactive*: Students can interpret the meaning of the average rate of change for each model provided over a specified domain. *Iconic*: Students can show the average rate of change over a specified domain on a table and graph (e.g., the secant lines). *Symbolic*: Students can calculate and interpret the average rate of change over a specified domain from an equation, table and graph.	
Teacher Notes:	The next unit, What Goes Up Must Come Down, will expand upon all these activities with quadratic functions.
Generalization Connection(s):	Mathematicians compare average rates of change over a specified interval to determine the increase or decrease of a function relative to another function.
Teacher Resources:	http://www.mrssnowsmath.com/algebra2.html (worksheets for parent functions) http://www.utdanacenter.org/highered/alg2/downloads/IV-B-CourseContentAlgII/AlgII_1-3-4.pdf (card matching activity from the

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https://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec2_mod4_funfeatures_se_83113.pdf (section4.7 provides resources related to function families)</td></tr>
<tr><td>Student Resources:</td><td colspan="2">http://www.projectsharetexas.org/resource/determining-parent-functionsontrack-algebra-1-module-2-lesson-1 (graphs of the parent functions)
http://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Parent_Functions_Cheat_Sheet_2014.pdf (descriptions of the parent functions)
http://www.ontrack-media.net/algebra2/A2M3L1ChartKey.pdf (descriptions of the parent functions)
https://www.itutoring.com/video/common-parent-functions (video of the parent functions)</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this lesson should be able to answer questions such as:
• How is the average rate of change represented in the graph, table and equation of various function families?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>https://wvde.state.wv.us/strategybank/FrayerModel.html (Frayer model template for vocabulary words)</td><td>Students can list points they are using in a table to help keep track of the x and y values and draw the secant lines on the graphs with vocabulary support.</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://www.ciclt.net/ul/okresa/Math%201%20Unit%201%20Function%20Families.pdf (unit on characteristics of function families)</td><td>Students can compare the properties of two function families verbally, algebraically, graphically, and numerically and create generalizations about the properties of each function family.</td></tr>
<tr><td>Key Knowledge and Skills:</td><td colspan="2">• Calculate and interpret the average rate of change of an exponential function (presented symbolically or as a table) over a specified interval and estimate the rate of change from a graph
• Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions</td></tr>
<tr><td>Critical Language:</td><td colspan="2">Key features, graphs, tables, relationships, calculate, interpret, compare, graphically, numerically, verbal descriptions, logarithms, exponential, functions, properties, transformations, average rate of change, classes of functions, radicals, rational, square root, cube root, piece-wise-defined functions, step functions, absolute value functions</td></tr>
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