Colorado's District Sample Curriculum Project

Colorado Teacher-Authored Instructional Unit Sample

Mathematics
5th Grade

Unit Title: Fraction Reaction

INSTRUCTIONAL UNIT AUTHORS

Clear Creek School District
- Kate Foy
- Graydon Harn
- Linda Mohrmann
- Michael Rossino

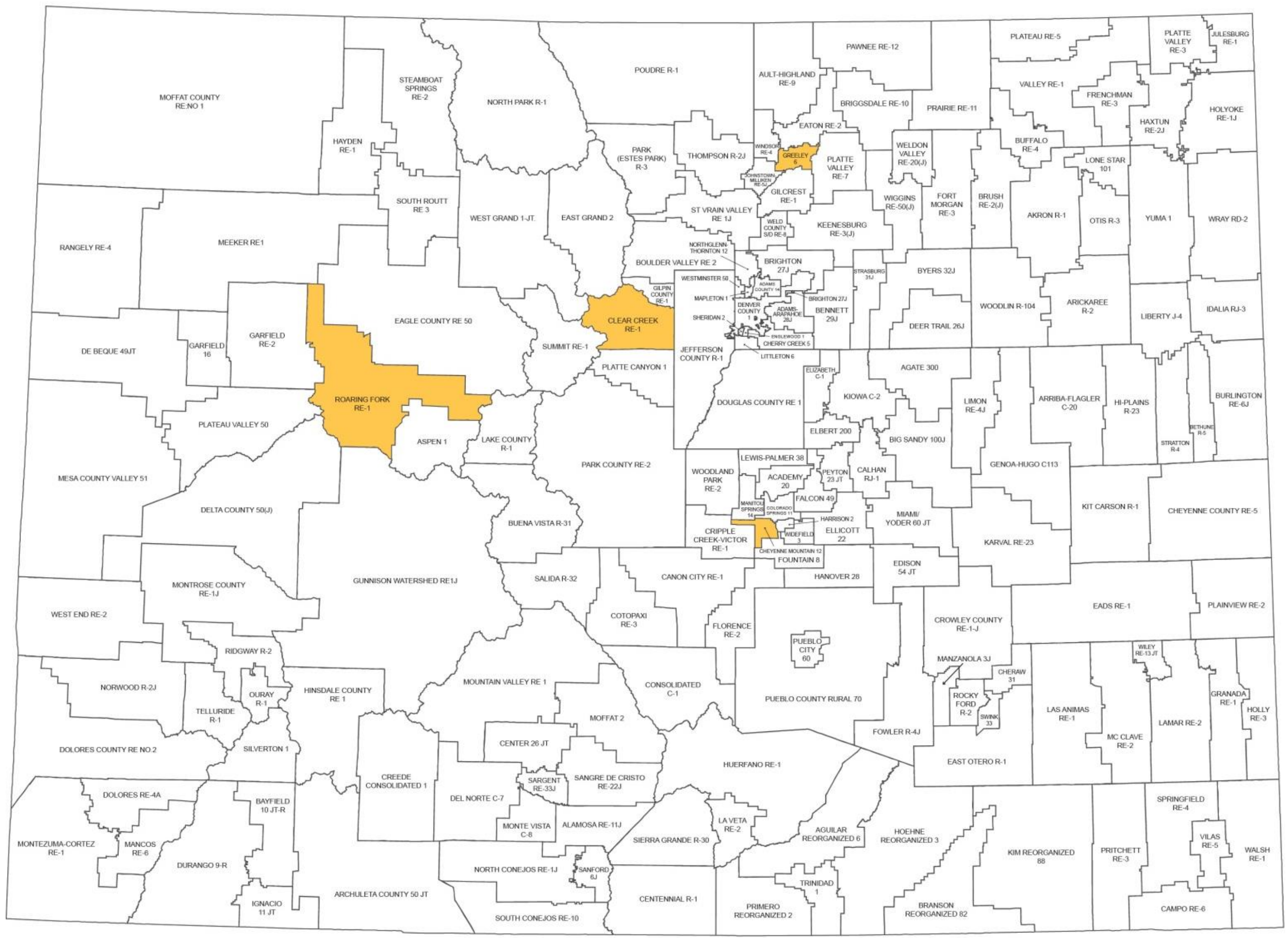
Greeley School District
- Jinny Andrews
- Michelle Harr-Anderson
- Cari Pettyjohn
- Kimberly Rudolph

BASED ON A CURRICULUM OVERVIEW SAMPLE AUTHORED BY

Cheyenne Mountain School District
- Kimberly Cumming

Clear Creek School District
- Brittney Huey

Roaring Fork School District
- Christine Horch



This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.

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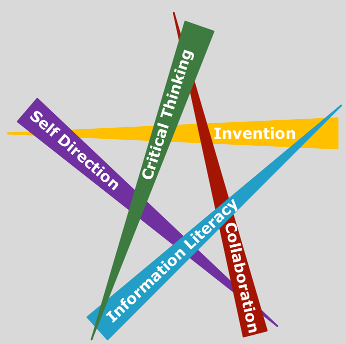
Colorado Teacher-Authored Sample Instructional Unit

Content Area	Mathematics	**Grade Level**	5th Grade
Course Name/Course Code			

Standard	**Grade Level Expectations (GLE)**	**GLE Code**
1. Number Sense, Properties, and Operations	1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms	MA10-GR.5-S.1-GLE.1
	2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency	MA10-GR.5-S.1-GLE.2
	3. Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency	MA10-GR.5-S.1-GLE.3
	4. The concepts of multiplication and division can be applied to multiply and divide fractions	MA10-GR.5-S.1-GLE.4
2. Patterns, Functions, and Algebraic Structures	1. Number patterns are based on operations and relationships	MA10-GR.5-S.2-GLE.1
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to interpret data	MA10-GR.5-S.3-GLE.1
4. Shape, Dimension, and Geometric Relationships	1. Properties of multiplication and addition provide the foundation for volume an attribute of solids	MA10-GR.5-S.4-GLE.1
	2. Geometric figures can be described by their attributes and specific locations in the plane	MA10-GR.5-S.4-GLE.2

Colorado 21st Century Skills



Critical Thinking and Reasoning: *Thinking Deeply, Thinking Differently*

Information Literacy: *Untangling the Web*

Collaboration: *Working Together, Learning Together*

Self-Direction: *Own Your Learning*

Invention: *Creating Solutions*

Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Titles	**Length of Unit/Contact Hours**	**Unit Number/Sequence**
Fraction Reaction	10 weeks	2

Unit Title	Fraction Reaction	**Length of Unit**	10 weeks
Focusing Lens(es)	Interpretation Relationships	**Standards and Grade Level Expectations Addressed in this Unit**	MA10-GR.5-S.1-GLE.3, MA10-GR.5-S.1-GLE.4
Inquiry Questions (Engaging-Debatable):	• How do operations with fractions compare to operations with whole numbers? (MA10-GR.5-S.1-GLE.3-IQ.1) • Why are there more fractions than whole numbers? (MA10-GR.5-S.1-GLE.3-IQ.2)		
Unit Strands	Number and Operations – Fractions		
Concepts	Denominator, numerator, fraction, addition, subtraction, common denominators, multiplication, division, whole number, unit fraction, expressions, area model, 1X1 unit square, partitioning, representation, rectangular regions, product, quotient, scaling (resizing), comparison, factor, estimation, real world problems, contexts, equal groups, fair sharing, rates, measurement (quotative), arrays, area,		

Generalizations **My students will Understand that…**	**Guiding Questions** **Factual**	**Conceptual**
The addition and subtraction of fractions necessitates common denominators in order to join or separate same size parts in the numerators of the fractions (MA10-GR.5-S.1-GLE.3-EO.a.i, a.ii)	What does the denominator of a fraction describe? How do you add or subtract fractions with different denominators? How can visual models be used represent and solve addition and subtraction of fraction problems involving unlike denominators? How can equations be used represent and solve addition and subtraction of fraction problems involving unlike denominators?	Why does 2/3 + 3/4 not equal 3/6? When adding fractions with a common denominator why does the denominator stay the same? Why do you need equivalent fractions when adding or subtracting? Why is it important to use benchmark fractions and number sense to estimate mentally the sums and differences of fractions?
The rewriting of an equation that multiplies a fraction by a whole number as a combination of whole number multiplication and division creates an equivalent equation (MA10-GR.5-S.1-GLE.4-EO.c)	How can you rewrite (3/4) x 5 as an expression involving multiplication and division of whole numbers?	Why is helpful to interpret multiplication of fractions by whole numbers as multiplication and division of whole numbers?
The calculation of the area of a rectangle with fractional lengths, as an extension of *l* x *w* = *A* for whole numbers, requires the usage of appropriate units of measure and the understanding of common factors/divisors (MA10-GR.5-S.1-GLE.4-EO.d, d.i)	How is the product of two fractions equivalent to the product of the numerators and denominators, based on an area model of fraction multiplication? How do you use common factors or common divisors in calculating fractional area?	Why is it not necessary to find a common denominator prior to multiplying two fractions?

Multiplication as scaling (resizing) rather than repeated addition allows mathematicians to compare the size of a product to the size of one factor on the basis of the other factor (MA10-GR.5-S.1-GLE.4-EO.e.i, e.ii)	What is different about stretching something that is 2 units long to be 3 times its original length (3x2) versus stretching something that is 1/3 a unit long to be 1/2 of its original length (1/2 x 1/3) ? What is the effect of a multiplying a given number by a fraction less than 1? What happens to the product if one of the factors is equivalent to a quantity of one? If we continued multiplying by smaller and smaller fractions, such as 1/5 × ½, 1/6 × ½, etc., what happens to the size of the products?	How can you predict the relative size of a product based on its factors? Why is it helpful to predict the relative size of a product? Why doesn't multiplication always make quantities larger? (MA10-GR.5-S.1-GLE.4-IQ.1)
Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area (MA10-GR.5-S.1-GLE.4-EO.f, i)	What is an example of multiplication of fraction problem involving equal groups? Fair sharing? Scaling? Area? Quotative division?	How are contexts involving whole number multiplication similar and different than those involving fractions?
The ability to multiply two fractions and to change their multiplication into expressions of whole number multiplication and division creates the foundation for solving division of whole numbers by a unit fraction and vice versa (MA10-GR.5-S.1-GLE.4-EO.g, h)	How can you change a unit fraction divided by a whole number problem to a missing factor multiplication problem to help you find the quotient? How can you change a whole number divided by a fraction problem to a missing factor multiplication problem and then change the multiplication problem into a string of whole number multiplication and division to help you find the quotient?	How does interpreting contextualized problems provide a foundation for understanding fraction division?
Fraction a/b (in which a is divided by b) can represent a fair share problem where a objects are shared by b people (MA10-GR.5-S.1-GLE.4-EO.a, b)	How can you share 5 cakes among 4 people? How can you share 5 cakes among 3 people?	How can you interpret a fraction as a fair share division problem when the fraction is greater than one? Less than one? Why doesn't division always make quantities smaller? (MA10-GR.5-S.1-GLE.4-IQ.2)

Key Knowledge and Skills: **My students will…**	*What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.*

- Add and subtract with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators (MA10-GR.5-S.1-GLE.3-EO.a.ii)
- Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers (MA10-GR.5-S.1-GLE.3-EO.a.i)
- Interpret a fraction as division of the numerator by the denominator (MA10-GR.5-S.1-GLE.4-EO.a)
- Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$ (MA10-GR.5-S.1-GLE.4-EO.c)
- Find the area of a rectangle with fraction side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths (MA10-GR.5-S.1-GLE.4-EO.d)
- Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas (MA10-GR.5-S.1-GLE.4-EO.d.i)
- Interpret multiplication as scaling (resizing) (MA10-GR.5-S.1-GLE.4-EO.e)
- Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication (MA10-GR.5-S.1-GLE.4-EO.e.i)
- Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relate the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1 (MA10-GR.5-S.1-GLE.4-EO.e.ii)
- Interpret division of a unit fraction by a non-zero whole number and a whole by number by a unit fraction (MA10-GR.5-S.1-GLE.4-EO.g, f)
- Compute quotients of a unit fraction by a non-zero whole number and whole number by a unit fraction (MA10-GR.5-S.1-GLE.4-EO.g, f)
- Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions (MA10-GR.5-S.1-GLE.3-EO.a.iii) and (MA10-GR.5-S.1-GLE.4-EO.b, f, i)

Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.
EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *"Mark Twain exposes the hypocrisy of slavery through the use of satire."*

A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):	*If I know the area of a piece of cloth is 6 square yards and one side is 1/5 of a yard I can find the other side by imagining the cloth cut into strips one-fifth wide and one yard long, this would give me five strips for every one square yard or a total of 30 strips which means the cloth is 30 feet long.*
Academic Vocabulary:	Addition, subtraction, multiplication, division, comparison, estimation, real world problems, equal groups, fair sharing,
Technical Vocabulary:	Numerator, denominator, benchmark fractions, improper fraction, mixed number, equivalent fraction, simplest form, whole number, unit fraction, expressions, product, quotient, scaling, resizing, factor, rates, measurement, arrays, area

Unit Description:	This unit focuses on the relationship between fractions and each of the four operations. The unit begins by developing the concept of a fraction as the operation of division through fair share problems. A major development in this unit is the formalization of multiplication of fractions. Students begin by modeling fraction multiplication using area and scaling models and eventually connect these models to the algorithm for multiplication. Students also explore division of fractions through contextualized problems and informal solution strategies: this concept is formalized and the algorithm is developed during sixth grade. Finally, students apply their understandings of multiplication of fractions and fraction equivalence to the addition and subtraction of fractions with unlike denominators.
Unit Generalizations	
Key Generalizations:	Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area
	The addition and subtraction of fractions necessitates common denominators in order to join or separate same size parts in the numerators of the fractions
Supporting Generalizations:	Fraction a/b (in which a is divided by b) can represent a fair share problem where “a” objects are shared by “b” people
	The calculation of the area of a rectangle with fractional lengths, as an extension of 1 x w = A for whole numbers, requires the usage of appropriate units of measure and the understanding of common factors/divisors
	Multiplication as scaling (resizing) rather than repeated addition allows mathematicians to compare the size of one factor on the basis of the other factor
	The rewriting of an equation that multiples a fraction by a whole number as a combination of whole number multiplication and division creates an equivalent equation

Performance Assessment: *The capstone/summative assessment for this unit.*	
Claims: (Key generalization(s) to be mastered and demonstrated through the capstone assessment.)	Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area. The addition and subtraction of fractions necessitates common denominators in order to join or separate same size parts in the numerators of the fraction.
Stimulus Material: (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)	As a writer of children’s literature, a publishing company has hired you to create a storybook about fractions for elementary students similar to the *Math Curse* by Jon Scieszka. Your story should include real world situations/scenarios about adding, subtracting, multiplying, and dividing fractions. Your story should include drawings and number sentences for each situation.
Product/Evidence: (Expected product from students)	Students will create a storybook with fraction scenarios. High quality books will include at least one fraction situation for each of the four operations. Each scenario should also include a visual representation (e.g., number lines, array models) and equation showing how to solve situation.
Differentiation: (Multiple modes for student expression)	Students can write their book in the form of a narrative, comic book, or picture book. Students can choose fractions ranging from benchmark fractions to mixed numbers. Students may work together to create one book, each writing and/or illustrating at least one scenario for each of the operations.

Texts for independent reading or for class read aloud to support the content

Informational/Non-Fiction	Fiction
	Math Curse by Jon Sciezka (Lexile level 560) *Full House: An Invitation to Fractions* by Dayle Ann Dodds (Lexile level 640) *If You Were a Fraction* by Trisha Speed Shaskan (Lexile level 620) *A Bear Fair Share* by Stuart J. Murphy (Lexile level 370) *Working with Fractions* by David A. Adler (Lexile level 690) *Fraction Action* by Loreen Leedy (Lexile level 330)

Ongoing Discipline-Specific Learning Experiences

1.	Description:	Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others [Mathematical Practice 3]	Teacher Resources:	http://schools.nyc.gov/NR/rdonlyres/D0A70F2D-1133-418C-B68F-95E6D714F357/0/NYCDOEG5MathStuffedwithPizza_Final.pdf (lesson plans contains exemplars that could be replicated for students to critique the reasoning of others)
			Student Resources:	N/A
	Skills:	Construct and communicate a complete and concise response, justify a conclusion using correct vocabulary, interpret and critique the validity of other's conclusions and reasoning, and identify errors and present correct solutions	Assessment:	Students analyze and defend their solutions for each major learning experience. Careful attention should be paid to precise use of vocabulary and symbols. Periodically throughout the unit, students could be provided with flawed solutions and asked to identify, describe, and correct the flaw.
2.	Description:	Think/work like a mathematician – Engaging in the practice of modeling the solution to real world problems [Mathematical Practice 4]	Teacher Resources:	https://www.sites.google.com/a/cmpso.org/caccss-resources/k-8-modeling-task-force/k-8-modeling-resources (examples of modeling problems and resources for teachers on teaching and scoring them) http://www.insidemathematics.org/index.php/standard-4 (video examples of students modeling with mathematics) http://learnzillion.com/lessons/1722-solve-multistep-word-problems-using-model-drawing (video about modeling)
			Student Resources:	N/A
	Skills:	Devise a plan to apply mathematics to solve everyday problems by using and making stated assumptions and approximations to simplify a	Assessment:	Modeling Problems Students use visual and symbolic fraction models such as number lines, bar models and array/area models to represent and analyze relationships of real world problems to draw conclusions and interpret results in relation to the context of the problem.

		real-world situation. Map relationships between important quantities by selecting appropriate tools to create models. Analyze mathematical relationships between important quantities to draw conclusions. Determine if the results make sense. If necessary, change the model		
3.	Description:	Mathematicians fluently multiply whole numbers and decimals.	Teacher Resources:	http://melrose.patch.com/groups/margaret-adamss-blog/p/bp--activities-at-home-for-increasing-math-fluency (recommendations to support fluency) http://www.mathwire.com/numbersense/bfacts.html (math games for the classroom to build fluency with basic facts)
			Student Resources:	http://www.multiplication.com/games (compilation of fact games) http://www.coolmath.com/prealgebra/02-decimals/decimals-cruncher-multiplication.htm (decimal multiplication)
	Skills:	Multiply whole numbers and decimals to the nearest hundredth.	Assessment:	Fluency Problems Students build fluency with the multiplication algorithm by practicing basic facts, multi-digit multiplication, decimal multiplication and accurately and quickly assessing the reasonableness of products.

Prior Knowledge and Experiences

Student familiarity with fractions as quantity that can be composed and decomposed and modeled on a number line will provide a strong foundation for this unit. Students will also benefit from prior experience with modeling multiplication with an area model for whole numbers will support the extension of this model to fractions within this unit.

Learning Experience # 1

The teacher may provide contexts for students to fair share a provided number of objects among a provided number of people so that students can explore the connection between division and fractions.

Enactive: Students can demonstrate how to fair share sub sandwiches by cutting paper representing subs (e.g., 5 subs for 4 people; 8 subs for 7 people; 5 subs sandwiches by 3 people; and 3 sub sandwiches by 4 people). Students can verbalize their answer to a partner (e.g., Each person gets 3/4 of a sub sandwich).
Iconic: Students can draw rectangles to represent the sub sandwiches and draw lines showing multiple ways to fair share the sub sandwiches (see teacher note).
Symbolic: Students can represent fair sharing with a division expression and corresponding fraction (e.g. 4 subs ÷ 7 people = 4/7 of a sub per person)

Teacher Notes:	When students begin to cut apart sub sandwiches they do it in a variety of ways. For instance when they share 3 subs among 4 people they might first give everyone half of a sub and then a fourth of a sub so each person gets 1/2 and 1/4 of a sub. During the class discussion it will be important to emphasize the equivalence of 1/2 and 1/4 to 3/4. Over the course of the enactive and iconic stages it is helpful to continually connect how the quantity each person receives is the same as the fraction formed by dividing the sub by the number of people. Student might see this in two ways. Some might first cut all of the subs into pieces equal to the number of people and each person gets one piece from each sub (e.g., if there are 3 subs and 4 people each sub is cut into four pieces and each person gets a piece from each sub 1/4 + 1/4 + 1/4) while some students immediately see that each person will receive the fraction formed by dividing the subs by the number of people (e.g., 3/4 of a sub per person). Discussions about each of these strategies over the course of the learning experience using a variety of problems can help students understand the generalization. It may also be helpful to ask students to try a strategy different from their initial strategy to allow them to make connections between the strategies. Throughout this discussion it is important to emphasize that the fractions relate to one sub and not the total amount of food. This is a common misconception among students solving this problem and can be alleviated if the students are asked to determine which group of people will get the most or least amount of food. By requiring students to make comparisons among the scenarios it highlights the need for the "whole" to remain constant.	
Generalization Connection(s):	Fraction a/b (where a is divided by b) can represent fair share problem where a objects are shared by b people Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area	
Teacher Resources:	http://www.contextsforlearning.com/samples/4_6SampleSession.pdf (description of a submarine sandwich lesson) http://www.engageny.org/resource/grade-5-mathematics-module-4 (examples of a lessons with fair share problems, Module 4, Topic B) http://learnzillion.com/lessonsets/65-understand-fractions-as-a-division-of-the-numerator-by-the-denominator (videos exploring fractions as division)	
Student Resources:	N/A	
Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: What are two different ways of fairly sharing 3 sandwiches among 4 people? Do people get the same amount of food using either strategy? Which group of people got the most food per person? How do you know? How does a fraction relate to division? Why can division be represented as a fraction? If each person receives 3/4, what does the 3/4 refer to all of the subs or one sub? Why? Why is it important to provide units (e.g., labels) when working with any number but particularly fractions?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide less complex scenarios of sharing (e.g., 2 subs by 2 people, 4 subs by 2 people, 2 subs by 4 people)	Students can determine how much food each person in a group will receive for less complex scenarios

Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide scenarios leading to fractions greater than one (e.g., 8 subs and 3 people)	Students can determine how much food each person in a group will receive for scenarios leading to fractions greater than one and explain why these scenarios still represent the division of the subs by the number of people
Key Knowledge and Skills:	• Interpret a fraction as division of the numerator by the denominator • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Equal groups, fair sharing, division, fraction, numerator, denominator, whole, quantity	

Learning Experience # 2	
The teacher may provide a variety of fractions (e.g., 2/3, 3/5, 1/8, 7/9) so that students can explore the creation of fair share story problems using these fractions. *Iconic*: Students can draw a picture representing each fraction as a fair share scenario and write a corresponding word problem. *Symbolic*: Students can exchange word problems and solve them writing the division expression and corresponding fraction in an equation format with appropriate units labels (e.g., 2 bags of marbles ÷ 3 people = 2/3 a bag of marbles per person).	
Generalization Connection(s):	Fraction a/b (where a is divided by b) can represent fair share problem where a objects are shared by b people Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area
Teacher Resources:	http://fractionbars.com/CommonCore/Gd5WkSh/CCSSDivWkSh5_NF_3(1).pdf (examples of story problems for fractions as division) http://fractionbars.com/CommonCore/Gd5WkSh/CCSSDivWkSh5_NF_3(2).pdf (examples of story problems for fractions as division) http://www.rcampus.com/rubricshowc.cfm?sp=yes&code=F2CC9B& (rubric for student created story problems) http://maccss.ncdpi.wikispaces.net/file/view/5thGradeUnit.pdf (unit on fraction multiplication and division with lessons on fair sharing)
Student Resources:	*Fractions: Making Fair Shares* by Michele Koomen
Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: Why does fair sharing connect to division? How does the word problem you created represent division and fair sharing? How can you interpret a fraction as a fair share division problem? Can you always fair share any number of objects with any number of people? Why or why not?

Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher can provide fair share word problems	Students can solve fair share word problems provided by the teacher using drawings and writing the corresponding division expression and fraction
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can create word problems for fractions greater than one (e.g., 11/4)
Key Knowledge and Skills:	• Interpret a fraction as division of the numerator by the denominator • Interpret a fraction as division of the numerator by the denominator • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Equal groups, fair sharing, division, fraction, numerator, denominator, whole, quantity	

Learning Experience # 3	
The teacher may provide the context of a story (e.g., multiplying jar) that multiplies or scales objects so that students can consider the impact of scaling a quantity by numbers greater than, less than, or equal to one. *Enactive*: Students can place 12 counters in a multiplying jar several times and find a new amount each time they open the jar (e.g., 48, 36, 24, 12, 9, 8, 6, 4, 3, 2, 1, 0). *Iconic*: Students can represent on a number line the number twelve and the amount they found in the jar and discuss with a partner if the value of multiplier is greater than, less than or equal to one.	
Teacher Notes:	It is important to set up the multiplying jar as always multiplying the quantity in it. If the teacher begins with whole number multipliers the students may be perplexed when they find less than twelve counters in the jar. This can lead to great discussion about the impact of multiplying a number by a quantity less than one. It may be helpful to have the jar produce products of 12 and 0 right before a product of 6. Initially students might want to guess that the jar divided by two but the jar always multiplies. During this learning experience students need only identify that the multiplier needs to be between 0 and 1. The next learning activity revisits this scenario and students are given the multipliers and asked to find the product. The teacher resources lists a video about scaling and multiplication using a flashlight context that may be helpful for students to watch and the student resource has an applet that shows the effect of scaling a frog by quantities less than, equal to or greater than one.
Generalization Connection(s):	Multiplication as scaling (resizing) rather than repeated addition allows mathematicians to compare the size of a product to the size of one factor on the basis of the other factor Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area
Teacher Resources:	http://learnzillion.com/lessons?utf8=%E2%9C%93&filters%5Bsubject%5D=math&query=5.NF.5&commit=Search+lessons (videos about scaling and multiplication, including flashlight video) *Eyes On Math: A Visual Approach to Teaching Math Concepts* by Marian Small

<table>
<tr><td></td><td colspan="2">http://www.illustrativemathematics.org/illustrations/22 (assessment task for multiplication as scaling)
http://www.illustrativemathematics.org/illustrations/164 (assessment task focusing on estimating the size of product based on concept of scaling)
http://www.illustrativemathematics.org/illustrations/49 (explores fraction multiplication with a calculator)</td></tr>
<tr><td>Student Resources:</td><td colspan="2">http://quadrivium.info/GGB/FrogMult.html (applet allows students to drag to different scale factors and see the impact on multiplying by scale factors greater than or less than one)
Anno's Magic Seeds by Mitsumasa Anno
Anno's Mysterious Multiplying Jar by Mitsumasa Anno</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this task should be able to answer questions such as:
When the answer in the multiplying jar was more than twelve what did you know about the multiplier?
When the answer in the multiplying jar was less than twelve what did you know about the multiplier?
What happens when you multiply a number by a quantity less than one?
Why can multiplication sometimes result in a quantity smaller than the initial amount?
What does it mean to scale by 1?
Does multiplying always result in an increase? Why or why not?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>N/A</td><td>Students can draw a picture to represent the original number of counters and result after being in the multiplying jar before showing it on a number line</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://momandmemath.blogspot.com/2013/04/5nf5-interpret-multiplication-as-scaling.html (multiplication as scaling using literature such as Clifford's Tricks by Norman Bridwell and Thumbelina by Hans Christian Anderson)</td><td>Students can explore scaling in literature such as Clifford's Tricks by Norman Bridwell and Thumbelina by Hans Christian Anderson to create a presentation about the impact of scaling the area of an object (e.g., doubling the length and width leads to an area four times greater)</td></tr>
<tr><td>Key Knowledge and Skills:</td><td colspan="2">• Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication
• Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relate the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1
• Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions</td></tr>
<tr><td>Critical Language:</td><td colspan="2">Multiply, multiplier, factor, product, scaling, resizing, fraction, times as many, times greater, times smaller, multiplier, quantity, whole, unit</td></tr>
</table>

Learning Experience # 4	
The teacher may provide the context of a story (e.g., multiplying jar) that multiplies or scales objects so that students can explore how to find product when they know the value of the multiplier. *Enactive*: Students can be told the multiplier for the jar (e.g., 2 times as many counters in the jar; 2/3 as many counters) and use counters to determine what will be in the jar after 12 counters are put into it. *Iconic*: Students can represent the multiplication they performed with counters on a double number line. For example, if the multiplier for the jar is 4, the student can draw 0, 1, 2, 3, 4 along the top of the number line and 0, 12, 2, 36, 48 along the bottom corresponding to each of the numbers along the top. If the multiplier for the jar is 3/4, the student can draw 0, 1/4, 2/4, 3/4, 1 along the top and write 0, 3, 6, 9, and 12 along the bottom of the number line. *Symbolic*: Students can write the multiplication equation representing the result of the multiplying jar (e.g., 12 X 3/4 = 9).	
Teacher Notes:	The teacher may want to begin with whole number multiplication and work through the iconic and symbolic portions of the learning experience for these examples before moving to fractional examples. It may also be helpful to run through the enactive, iconic, and symbolic parts of the lesson for a few unit fraction examples such as multiplying by 1/2, 1/3 and 1/4 before moving to more complex examples such as 2/3. This activity can be continued throughout the unit by changing the initial amount in the multiplication jar. It is important for students to have experience with scaling by one (e.g., 12 x 1). The concept of scaling by one will be built upon latter when working with equivalent fractions. Teachers may even choose to name the number one as 4/4 or 2/2 if students appear comfortable with these representations. It is also helpful to discuss with students that when scaling the "whole" might not be one. In the case of the multiplying jar the whole or referent group is 12. This is visible on the double number line where the number one always corresponds to the 12. Students may struggle with the double line. It is helpful to explicitly connect it to the work they do with the counters during the enactive stage.
Generalization Connection(s):	The rewriting of an equation that multiplies a fraction by a whole number as a combination of whole number multiplication and division creates an equivalent equation Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area
Teacher Resources:	http://www.math-aids.com/Fractions/Multiplying_Fractions_and_Whole_Numbers.html (worksheet creator for fractions multiplied by whole numbers) http://www.illustrativemathematics.org/illustrations/49 (explores fraction multiplication with a calculator) http://learnzillion.com/lessons?utf8=%E2%9C%93&filters%5Bsubject%5D=math&query=5.NF.5&commit=Search+lessons (videos about scaling and multiplication, including flashlight video)
Student Resources:	http://www.coolmath4kids.com/fractions/fractions-15-multiplying-whole-mixed-numbers-01.html (whole number multiplied by a fraction practice, with visuals) http://www.ixl.com/math/grade-7/multiply-fractions-and-whole-numbers (whole number multiplied by a fraction practice, without visuals) *Anno's Magic Seeds* by Mitsumasa Anno *Anno's Mysterious Multiplying Jar* by Mitsumasa Anno

Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: Which problems were hardest to solve? Why? How is scaling by a fraction different or the same as scaling by a whole number? How does multiplying by a unit fraction differ from multiplying by fractions with numerators greater than one? Why on your double number line was the number one opposite the number 12? What would happen if the initial amount put into the multiplying jar changed? Is the product of 12 X 1/2 different from 1/2 X 12? Why or why not?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can find the multiplication equation for the multiplying jar for whole numbers and unit fractions (e.g., 1/2, 1/3, 1/4)
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can find the multiplication equation for the multiplying jar for fractions greater than one (e.g., 11/4, 9/2)
Key Knowledge and Skills:	• Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$ • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Multiply, multiplier, factor, product, scaling, resizing, fraction, times as many, times greater, times smaller, multiplier, quantity, whole, unit, commutative	

Learning Experience # 5

The teacher may provide word problems containing situations where whole numbers are multiplied by fractions and vice versa so that students can begin developing efficient strategies for solving these types of problems.

Enactive: The student can use counters to represent and solve the word problems (e.g., If the problem is 8 X 3/4 the student can solve it by partitioning 8 counters into four equal partitions of 2 and then choosing three of the four partitions resulting in 6).

Iconic: The student can represent on a double number line how to solve the word problems (e.g., If the problem is 8 X 3/4 the student can draw 0, 1/4, 2/4, 3/4, 1 along the top and write 0, 2, 4, 6, and 8 along the bottom of the number line).

Symbolic: The student can write the multiplicative expression representing the word problem and equivalent expressions to represent how they solve the word problems (e.g., If the problem is 8 X 3/4 they might write 8 X 3/4 = 8 ÷ 4 X 3 or 8 X 3/4 = 8 X 1/4 X 3 or 8 X 3/4 = 8 X 3 ÷ 4 or 8 X 3 X 1/4). Students can then explain to a partner how their symbolic expression connects to how they solved the problem using either counters and/or the double number line.

Teacher Notes:	This learning experience builds on the fair share work from the first two learning experiences and the fraction work in fourth grade by extending the notion that every fraction can be seen as both a division and multiplication problem at the same time (e.g., 3/4 is both 3 ÷ 4 and 3 X 1/4). This flexible understanding of fractions supports the development of the symbolic method for

	multiplying fractions. As the problems become more complex and lead to fractional answers the enactive strategy of counters may not work for students which will prompt a need to focus more on drawings, number lines and equations. This can lean to a discussion about the need for symbolic methods for more complex problems. It is important to emphasize the benefit of always having a visual model in your head when solving a problem to ensure understanding of the symbols and procedures.	
Generalization Connection(s):	The rewriting of an equation that multiplies a fraction by a whole number as a combination of whole number multiplication and division creates an equivalent equation Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area	
Teacher Resources:	http://www.ixl.com/math/grade-5/multiply-fractions-by-whole-numbers-word-problems (word problems with fractions multiplied by whole numbers)	
Student Resources:	http://www.ixl.com/math/grade-5/multiply-fractions-by-whole-numbers-word-problems (word problems with fractions multiplied by whole numbers)	
Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: What strategies do you use to solve problems involving multiplication of whole number by a fraction? What is the most efficient strategy for solving multiplication of whole number by a fraction? Can you solve all problems involving multiplication of a whole number by a fraction using counters? Why or why not? How does understanding a fraction as both a division and multiplication problem help solve fraction multiplication more efficiently?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can solve word problems for fraction and whole number multiplication using only unit fractions and benchmark fractions that evenly partition the whole number
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can create word problems for fraction and whole number multiplication Students can solve world problems for fraction and whole number multiplication that result in fractions greater than one
Key Knowledge and Skills:	• Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$ • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Fractions, whole numbers, partition, equal groups, double number lines, numerator, denominator, multiplication, division, unit fraction	

Learning Experience # 6	
The teacher may provide fraction multiplication word problems so that students can begin developing efficient strategies for solving these types of problems. Enactive: Students can partition counters to solve fraction multiplication problems by choosing a quantity of counters and trying to take a fraction of fraction of the counters (e.g., students can find the product of 2/3 x 3/4 by starting with 24 counters and first finding 2/3 of 24/24 which is 16/24 and then 3/4 of 16/24 which is 12/24 or 1/2). *Iconic*: Students can partition a square to solve fraction multiplication problems by using the concept of area (e.g., students can find the product of 2/3 x 3/4 by first partitioning a square into three rows and shading two of them and then partitioning the same square into four columns and shading three of them, the portion of the square with overlapping shading, 6/12, is the answer). *Symbolic*: Students can work with partners to create a poster explaining the traditional algorithm for multiplying fractions based on the models from the enactive and/or iconic stages of the learning experience. Students can practice solving fraction multiplication problems with and without a context.	
Teacher Notes:	Students may struggle in the enactive stage finding a quantity of counters that will result in a whole number when finding a fraction of fraction of the counters. Once they find a quantity through trial and error they can apply their understandings and skills from the previous learning experiences about fraction and whole number multiplication to find the a fraction of a fraction of the counters. It is important to emphasize the answer in the enactive stage is the number of counters they found compared to original number of counters. Students should find this strategy inefficient but it provides a concrete way of visualizing the algorithm. For instance, students might notice they can partition the counters by both denominators first and then create groups of groups with both numerators, which connects to the symbolic algorithm of multiplying the denominators and numerators. Simplifying fractions is not necessary throughout this learning experience but it is important that students can recognize the equivalence of their answers. This can be done on a number line if it is not readily obvious to students.
Generalization Connection(s):	The rewriting of an equation that multiplies a fraction by a whole number as a combination of whole number multiplication and division creates an equivalent equation Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area
Teacher Resources:	http://www.ixl.com/math/grade-5/multiply-two-fractions-word-problems (word problems with fractions multiplied by fractions) http://learnzillion.com/lessons/125-multiply-fractions-by-fractions-finding-a-part-of-a-part (video about finding fractions of fractions) http://learnzillion.com/lessons/213-multiply-fractions-by-fractions-using-area-models (video about finding fractions of fractions using area models) http://www.illustrativemathematics.org/illustrations/49 (explores fraction multiplication with a calculator)
Student Resources:	http://www.ixl.com/math/grade-5/multiply-two-fractions-word-problems (practice fraction times fraction story problems) http://www.mathplayground.com/fractions_mult.html (fraction multiplication without contexts) http://www.arcademics.com/games/snow-sprint/snow-sprint.html (fraction multiplication game)
Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: When using counters to multiply fractions what do the counters represent? What is the unit or whole when using counters to multiply fractions? What is the unit or whole when using an area model to multiply fractions? When multiplying fractions what do the denominators represent? What multiplying fractions what do the numerators represent?

	How does the area model help explain the traditional algorithm for multiplication of fractions? Why does the traditional algorithm for multiplication of fractions work? How does multiplication of whole numbers and fractions provide the foundation for solving fraction multiplication problems?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://www.cpalms.org/Public/PreviewResource/Preview/38004 (explanation of fractions times fractions using an area model)	Students can solve fraction multiplication problems scaffolded by an explanation of the area model
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can develop an enactive, iconic and symbolic model for solving fraction multiplication involving mixed numbers
Key Knowledge and Skills:	• Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$ • Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Fractions, whole numbers, partition, equal groups, numerator, denominator, multiplication, division, unit fraction, area	

Learning Experience # 7	
The teacher may provide a variety of equivalent fractions so that students can explore the role of the number one in creating equivalent fractions. *Iconic*: Students can compare two equivalent fractions by representing both on squares and then finding a way to partition the squares to show the same quantities. *Symbolic*: Students can explain why partitioning the squares was the same as multiplying by a fraction equivalent to one by connecting to the area model of the square and the traditional algorithm for multiplication developed in the previous learning experience. Students can then practice building additional equivalent fractions.	
Teacher Notes:	During the class discussion it may be helpful to discuss the multiplication identity property of the number one (e.g. 1 x 12 = 12 so 1 x ¾ = ¾) and its importance in working with fractions (e.g., 3/4 x 2/2 = 6/8).
Generalization Connection(s):	Multiplication as scaling (resizing) rather than repeated addition allows mathematicians to compare the size of a product to the size of one factor on the basis of the other factor Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area
Teacher Resources:	http://learnzillion.com/lessons/3440-understand-multiplication-by-a-fraction-equivalent-to-one (video demonstrating multiplication by a fraction equivalent to one) http://learnzillion.com/lessons/3409-understand-the-effect-of-multiplying-by-a-fraction-equal-to-one (video explaining multiplication by a fraction equivalent to one)

Student Resources:	N/A	
Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: Is the more than one way to multiply by the number one? How do you know a fraction is equal to one? How can you represent 1 as a fraction? Why is multiplying by the number one special? How is multiplying by the number one similar to adding by zero?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://www.printable-math-worksheets.com/support-files/manipulative-fraction-strips.pdf (blackline master of fraction strips)	Students can find equivalent fractions using fraction strips
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can compare non-equivalent fractions by finding common denominators
Key Knowledge and Skills:	• Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relate the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1 • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Identity, fractions, whole numbers, partition, equal groups, numerator, denominator, multiplication, area model, one, equivalent	

Learning Experience # 8	
The teacher may provide a variety of fraction multiplication expressions (e.g., 1/2 x 2/3) so that students can use what they know about scaling to predict size of product. *Iconic*: Students can locate the two factors of the multiplication problem on a number line and then show where the answer will be located in relation to one of the two factors (i.e., if you are multiplying 2/3 X 3/4 since both fractions are less than one you can shade the number line from 2/3 to zero). *Symbolic*: Students can create examples of multiplication problems with fractions greater than, less than, and equal to one and write an explanation of how to predict the size of the product for each.	
Teacher Notes:	Teachers may provide students with fractional multiplication problems which are extremely difficult because students are not required to solve the problems in this learning experience rather the goal is to help them determine the relative size of the product compared to the factors. It may be helpful to remind students of the idea of scaling from prior learning experiences as a meaning for multiplication if they are struggling to make predictions.

Generalization Connection(s):	Multiplication as scaling (resizing) rather than repeated addition allows mathematicians to compare the size of a product to the size of one factor on the basis of the other factor Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area	
Teacher Resources:	http://learnzillion.com/lessons/3440-understand-multiplication-by-a-fraction-equivalent-to-one (a video for teachers and potentially students to watch to demonstrate multiplication by a fraction equivalent to one) http://learnzillion.com/lessons/3409-understand-the-effect-of-multiplying-by-a-fraction-equal-to-one (a video for teachers and potentially students to watch to demonstrate multiplication by a fraction equivalent to one)	
Student Resources:	N/A	
Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: How does the concept of scaling help make predictions about the size of a product? Do all fractions result in products smaller than the original number? Why or why not? Why is it important to make predictions about the size of a product?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can make predictions for whole numbers multiplied by fractions greater than, equal to, or less than one
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can generate a story problem to reflect a given equation
Key Knowledge and Skills:	• Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relate the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1 • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Product, predict, identity, fractions, multiplication, number line, equivalent, factors	

Learning Experience # 9

The teacher may provide contexts for finding the area of rectangles with fractional sides and sides with mixed numbers so that students can connect their understanding of fractional multiplication to area.

Iconic: The student can use graph paper to model and solve fractional area problems.

Symbolic: The student can represent area multiplication word problems with multiplication equations.

<table>
<tr><td>Teacher Notes:</td><td colspan="2">Students need to use both fractional sides and sides with mixed numbers in order to show mastery of the ideas in this learning experience. For instance, it may help to start with a problem such as 5 ½ inches X 3 ½ inches. Students can notice when drawing the rectangle on grid paper that they have fractions of squares. It is important to discuss the size of the fractional pieces and relate them back to their understanding of fractional multiplication. Students might notice that the answer to this problem involves four separate problems 5 x 3, 5 x 1/2, 3 x 1/2, and 1/2 x 1/2. This use of the distributive property can be connected to their work with multi-digit whole numbers and the area model. A common misconception of students is that mixed numbers represent multiplication rather than addition. It may help students to emphasize to students that 5 ½ is actually 5 + ½ the addition sign is just hidden. This connection to addition can help students when modeling the problems and when connecting the models to the distributive property.</td></tr>
<tr><td>Generalization Connection(s):</td><td colspan="2">The calculation of the area of a rectangle with fractional lengths, as an extension of l x w = A for whole numbers, requires the usage of appropriate units of measure and the understanding of common factors/divisors
Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area</td></tr>
<tr><td>Teacher Resources:</td><td colspan="2">http://learnzillion.com/lessons/1545-find-the-product-of-a-whole-number-and-a-fraction-by-using-area-models (video demonstrating area of a rectangle with whole numbers and fractional sides)
http://learnzillion.com/lessons/1542-find-the-area-of-a-rectangle-with-fractional-side-lengths-by-tiling (video demonstrating area of a rectangle with fractional sides)
http://learnzillion.com/lessons/1546-find-the-product-of-a-fraction-and-a-mixed-number-by-using-area-models (video demonstrating area of a rectangle with mixed numbers)
http://www.printablepaper.net/category/graph (free downloadable graph paper)</td></tr>
<tr><td>Student Resources:</td><td colspan="2">N/A</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this task should be able to answer questions such as:
How is finding the area of rectangles with whole number since lengths similar and different from find the area of rectangles with fractional side lengths?
How is multiplying multi-digit whole numbers similar to multiplying mixed numbers?
How can you predict area based on the side lengths?
Why is area measured in squares?
What is a square one-half by one-half only one-fourth of a square inch and not one-half of a square inch?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>The teacher may provide fractional area multiplication problems with the corresponding area model already drawn</td><td>Students can solve fractional area multiplication problems using drawings provided by the teacher</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>N/A</td><td>Students can show how to multiply mixed numbers by changing them to improper fractions</td></tr>
</table>

Key Knowledge and Skills:	• Find the area of a rectangle with fraction side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths • Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions
Critical Language:	Fractions, whole numbers, mixed numbers, multiplication, area

Learning Experience # 10

The teacher may revisit the scaling context (e.g., multiplying jar) so that students can explore the connections between multiplication and division with whole numbers and unit fractions.

Enactive: Students can be provided counters representing the final amount in the jar and be given a unit fraction multiplier for the jar and asked to find the initial amount using counters.

Iconic: Students can represent the multiplication/division problem from the multiplying jar on a double number line. For example, if the jar ends with 12 counters and the multiplier is 4, students can label the bottom of the number line 0, 1, 2, 3, 4 and label the top of the number line 0, 3, 6, 8, 12 to find that initial amount or one value is 3. When the jar contains 2 counters and the multiplier is 1/5, students can label the bottom part of the number line 0, 1/5, 2/5, 3/5, 4/5, 5/5 and label the top of the number line 0, 2, 4, 6, 8, 10 corresponding to each number along the bottom, to find the initial amount is 10.

Symbolic: Students can write the multiplication and corresponding division equation representing the result of the multiplying jar (e.g., 10 X 1/5 = 2 and 2 ÷ 1/5 = 10).

Teacher Notes:	The teacher may want to begin with whole number multipliers and work through the iconic and symbolic portions of the learning experience before moving to unit fraction examples. Through the learning experience it is helpful to reinforce what each quantity means in relation to the context of the problem (e.g., initial amount, multiplier, final amount). Students might begin to notice that the answer is always the same as the denominator of the multiplier multiplied by the final amount when working with unit fractions. This is the foundation for eventually understanding the algorithm for dividing fractions. It is helpful if students recognize this pattern to ask them to justify it but the formalization of division of fractions should wait till sixth grade.
Generalization Connection(s):	The ability to multiply two fractions and to change their multiplication into expressions of whole number multiplication and division creates the foundation for solving division of whole numbers by a unit fraction and vice versa Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area
Teacher Resources:	http://learnzillion.com/lessons?utf8=%E2%9C%93&filters%5Bsubject%5D=math&query=5.NF.5&commit=Search+lessons (videos about scaling and multiplication, including flashlight video)
Student Resources:	http://www.coolmath4kids.com/fractions/fractions-15-multiplying-whole-mixed-numbers-01.html (whole number multiplied by a fraction practice, with visuals) http://www.ixl.com/math/grade-7/multiply-fractions-and-whole-numbers (whole number multiplied by a fraction practice, without visuals) *Anno's Magic Seeds* by Mitsumasa Anno *Anno's Mysterious Multiplying Jar* by Mitsumasa Anno

Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: How did your strategy change when the multiplier for the jar was a whole number versus a fraction? Which was easier to solve? Why? How are multiplication and division related in the multiplying jar? What other division problems could be created using the multiplying jar? (i.e., find the multiplier if we know the initial and final amount) What patterns did you notice when you were solving for the initial amount in the jar?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide a double number line template for each problem with the bottom or multiplier part filled out	Students can find the initial amount in the jar by completing a partially filled out double number line
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can create story problems similar to the multiplying jar with unit fraction multipliers and known products
Key Knowledge and Skills:	• Interpret division of a unit fraction by a non-zero whole number and a whole by number by a unit fraction • Compute quotients of a unit fraction by a non-zero whole number and whole number by a unit fraction • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Division, multiplication, multiplier, factor, product, scaling, resizing, fraction, quantity, whole, unit, unit fraction	

Learning Experience # 11	
The teacher may provide word problems involving the division of a unit fraction by a whole number so that students can explore the connections between multiplication and division with unit fractions and whole numbers. *Iconic*: Students can use visual representations (e.g. drawings, pictures of objects, number lines) to solve word problems (e.g., Jane has ¼ of a pan of banana bread. She wants to share this between herself and 2 friends. How much banana bread does each person get?). *Symbolic*: Students can write the division equation for each word problem and explain the meaning of each part of the equation (e.g., 1/4 ÷ 2 = 1/8 means 1/4 of a pan of banana bread shared among 2 people means each person gets 1/8 of a pan of banana bread) and explain the corresponding multiplication problem (2 x 1/8 = 1/4 means 2 people each get 1/8 of a pan of banana bread which is 1/4 of pan of banana bread).	
Teacher Notes:	A common misconception is for students to divide the fractional piece but lose focus on the whole (e.g. when splitting 1/4 ÷ 2 = 1/8 not 1/2). Students will continue to explore the idea of division of fractions in sixth grade.

Generalization Connection(s):	The ability to multiply two fractions and to change their multiplication into expressions of whole number multiplication and division creates the foundation for solving division of whole numbers by a unit fraction and vice versa Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area	
Teacher Resources:	http://www.illustrativemathematics.org/5.NF.B.7 (sample assessment tasks for word problems involving division of whole numbers by unit fractions and vice versa) http://learnzillion.com/lessons/3787-divide-a-unit-fraction-by-a-whole-number (video to demonstrate division of a unit fraction by a whole number) http://learnzillion.com/lessons/130-divide-unit-fractions-drawing-pictures (video to demonstrate division of a unit fraction by a whole number) http://learnzillion.com/lessons/3767-divide-a-whole-number-by-a-unit-fraction (video to demonstrate division of a whole number by a unit fraction) http://learnzillion.com/lessons/131-divide-by-unit-fractions-using-the-number-line (video to demonstrate division of a whole number by a unit fraction using a number line)	
Student Resources:	http://www.ixl.com/math/grade-5/divide-fractions-by-whole-numbers-word-problems (practice word problems of dividing fractions by whole numbers)	
Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: How does the division of a fraction by whole numbers differ from the division of whole numbers? Why is it important to think about the whole while dividing fractions? How are multiplication and division related?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide a model for solving a unit fraction divided by a whole number problem (http://fractionbars.com/CommonCore/Gd5Les/CCSSDiv5_NF_7aGd5.pdf)	Students can solve a unit fraction divided by a whole number problem by referring to a model problem
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can create division problems involving fractions divided by whole numbers
Key Knowledge and Skills:	• Compute quotients of a unit fraction by a non-zero whole number and whole number by a unit fraction • Interpret division of a unit fraction by a non-zero whole number and a whole by number by a unit fraction • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Division, multiplication, multiplier, factor, product, fraction, quantity, whole, unit, quotient, unit fraction	

Learning Experience # 12	
The teacher may provide word problems involving the addition and subtraction of fractions with unlike denominators (e.g., Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the other needed 2/3 cup of sugar. How much sugar did he need to make both recipes?) so that students can explore the importance of equivalent fractions when composing and decomposing fractions. *Enactive*: Students can use fraction strips to solve fraction addition and subtraction word problems with unlike dominators and explain to a partner why 1/2 + 1/3 does not equal 2/5 using fraction strips. *Iconic*: Students can use area models to create common denominators to solve word problems involving the addition and subtraction of fractions with unlike denominators and explain why common denominators are necessary. *Symbolic*: Students can solve word problems involving the addition and subtraction of fractions with unlike denominators by using the multiplication of numbers equivalent to one (e.g., 2/2, 3/3) to create common denominators and assess the reasonableness of their answer by using a number line.	
Teacher Notes:	The standards in fourth grade provide the basis for adding and subtracting fractions with common denominators. Fifth grade adds the concept of unlike denominators and connects it to the idea of equivalent fractions and multiplication by one. Problems in this learning experience should include fractions larger than one including mixed numbers.
Generalization Connection(s):	The addition and subtraction of fractions necessitates common denominators in order to join or separate same size parts in the numerators of the fractions
Teacher Resources:	https://docs.google.com/document/d/1Up8jMPRlrx_wKzE3D_DJiY9T4oK9_GZdRcoYqRStqfw/edit (a document defining the differences between addition and subtraction problems) http://www.opusmath.com/common-core-standards/5.nf.1-add-and-subtract-fractions-with-unlike-denominators-including-mixed (a website page that displays many different word problems) http://learnzillion.com/lessons/2638-find-common-denominators-by-creating-area-models (video to demonstrate finding common denominators) http://learnzillion.com/lessons/2659-add-fractions-with-unlike-denominators-by-creating-area-models (video to demonstrate adding fractions with unlike denominators) http://learnzillion.com/lessons/2624-subtract-fractions-with-unlike-denominators-by-creating-area-models (video to demonstrate adding fractions with unlike denominators) http://learnzillion.com/lessons/2483-add-mixed-numbers-with-unlike-denominators-using-fraction-bars (video to demonstrate adding mixed numbers) http://learnzillion.com/lessons/2443-subtract-fractions-and-mixed-numbers-by-decomposing (video to demonstrate subtracting fractions and mixed numbers) http://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-fractions-topic (Khan academy addition and subtraction unit with videos and worksheets
Student Resources:	http://www.mathnook.com/math/brainracerfraction.html (adding and subtracting fraction game called Brain racer) http://www.mathnook.com/math/city-under-siege-fractions.html (adding and subtracting fraction game called City under siege) http://www.mathnook.com/math/geniusdefenderfractions.html (adding and subtracting fraction game called Genius defender) http://www.mathnook.com/math/math-balloons-fractions.html (adding and subtracting fraction game called Math balloons) http://www.mathnook.com/math/panda-runner-fraction.html (adding and subtracting fraction game called Panda Runner)

Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: Why is common denominator important when adding and subtracting fractions? When multiplying fractions do you find a common denominator? Why or why not? How can you use an area model to represent how to add and subtract fractions? How is addition of fractions similar or different from adding whole numbers? How is subtraction of fractions similar or different to subtracting whole numbers?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can solve word problems using benchmark fractions with unlike denominators referring to the same whole when adding and subtracting fractions
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can create word problems involving addition and subtraction of fractions
Key Knowledge and Skills:	• Add and subtract with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators • Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers • Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators • 5.NF.1 and 5.NF.2	
Critical Language:	Denominator, numerator, common, unlike, addition, subtraction, multiplication, equivalent fractions, sums, difference	

Learning Experience # 13	
The teacher may provide word problems, equations, and visual models (e.g., card sets) so that students can begin to recognize appropriate operations for word problems and make connections to equations and visual model.	
Teacher Notes:	It is helpful if the word problems have the same basic fractions so that students must read the problem to determine the right equation to match it to rather than simply looking at the numbers in the problem and guessing the appropriate match. The performance task for this unit asks students to write story problems for each of the four operations. This learning experience provides students with concurrent experiences involving fraction word problems for each of the four operations to support them in recognizing the variations in types of word problems.
Generalization Connection(s):	Real world problems for multiplication and division of fractions often involve contexts such as of equal groups, fair sharing, rates, measurement, scaling, and arrays/area
Teacher Resources:	http://www.jennyray.net/uploads/1/2/9/7/12975776/decomposing_numbers_operations_and_algebraic_thinking_kindergarten_spring_2012_revised_4-25-12.pdf (shows the template for a lesson matching addition equations to pictorial representations)
Student Resources:	http://www.ixl.com/math/grade-5 (practice with mixed operation word problems)

Assessment:	Students mastering the concept and skills of this task should be able to answer questions such as: How can you determine the difference between addition and multiplication problems? How can you determine the difference between subtraction and multiplication problems? How can you determine the difference between division and multiplication problems?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can match fewer cards at a time
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can solve the word problems by creating the equation and visual cards
Key Knowledge and Skills:	• Solve word problems using visual models or equations to represent the problem which involve: addition and subtraction of fractions referring to the same whole, including cases of unlike denominators; division of whole numbers leading to answer in the form of fractions or mixed numbers; multiplication of fractions and mixed numbers; and, division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions	
Critical Language:	Sum, difference, product, quotient, numerator, denominator, fraction, unit fraction, equation, matching, word problem	