



Colorado Academic S T A N D A R D S

Fifth Grade

Mathematics at les







Colorado Academic Standards in Mathematics and The Common Core State Standards for Mathematics

On December 10, 2009, the Colorado State Board of Education adopted the revised Mathematics Academic Standards, along with academic standards in nine other content areas, creating Colorado's first fully aligned preschool through high school academic expectations. Developed by a broad spectrum of Coloradans representing Pre-K and K-12 education, higher education, and business, utilizing the best national and international exemplars, the intention of these standards is to prepare Colorado schoolchildren for achievement at each grade level, and ultimately, for successful performance in postsecondary institutions and/or the workforce.

Concurrent to the revision of the Colorado standards was the Common Core State Standards (CCSS) initiative, whose process and purpose significantly overlapped with that of the Colorado Academic Standards. Led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), these standards present a national perspective on academic expectations for students, Kindergarten through High School in the United States.

Upon the release of the Common Core State Standards for Mathematics on June 2, 2010, the Colorado Department of Education began a gap analysis process to determine the degree to which the expectations of the Colorado Academic Standards aligned with the Common Core. The independent analysis proved a nearly 95% alignment between the two sets of standards. On August 2, 2010, the Colorado State Board of Education adopted the Common Core State Standards, and requested the integration of the Common Core State Standards and the Colorado Academic Standards.

In partnership with the dedicated members of the Colorado Standards Revision Subcommittee in Mathematics, this document represents the integration of the combined academic content of both sets of standards, maintaining the unique aspects of the Colorado Academic Standards, which include personal financial literacy, 21st century skills, school readiness competencies, postsecondary and workforce readiness competencies, and preschool expectations. The result is a world-class set of standards that are greater than the sum of their parts.

The Colorado Department of Education encourages you to review the Common Core State Standards and the extensive appendices at www.corestandards.org. While all the expectations of the Common Core State Standards are embedded and **coded with CCSS:** in this document, additional information on the development and the intentions behind the Common Core State Standards can be found on the website.

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Colorado Academic Standards Mathematics Standards

"Pure mathematics is, in its way, the poetry of logical ideas."

Albert Einstein

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"If America is to maintain our high standard of living, we must continue to innovate. We are competing with nations many times our size. We don't have a single brain to waste. Math and science are the engines of innovation. With these engines we can lead the world. We must demystify math and science so that all students feel the joy that follows understanding."

Dr. Michael Brown, Nobel Prize Laureate

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In the 21st century, a vibrant democracy depends on the full, informed participation of all people. We have a vast and rapidly growing trove of information available at any moment. However, being *informed* means, in part, using one's sense of number, shape, data and symbols to organize, interpret, make and assess the validity of claims about quantitative information. In short, informed members of society know and do mathematics.

Mathematics is indispensable for understanding our world. In addition to providing the tools of arithmetic, algebra, geometry and statistics, it offers a way of thinking about patterns and relationships of quantity and space and the connections among them. Mathematical reasoning allows us to devise and evaluate methods for solving problems, make and test conjectures about properties and relationships, and model the world around us.

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Standards Organization and Construction

As the subcommittee began the revision process to improve the existing standards, it became evident that the way the standards information was organized, defined, and constructed needed to change from the existing documents. The new design is intended to provide more clarity and direction for teachers, and to show how $21^{\rm st}$ century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The "Continuum of State Standards Definitions" section that follows shows the hierarchical order of the standards components. The "Standards Template" section demonstrates how this continuum is put into practice.

The elements of the revised standards are:

Prepared Graduate Competencies: The preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Standard: The topical organization of an academic content area.

High School Expectations: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate. What do students need to know in high school?

Grade Level Expectations: The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from preschool through eighth grade?*

Evidence Outcomes: The indication that a student is meeting an expectation at the mastery level. How do we know that a student can do it?

21st Century Skills and Readiness Competencies: Includes the following:

• Inquiry Questions:

Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

• Relevance and Application:

Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

• Nature of the Discipline:

The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

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Continuum of State Standards Definitions

Prepared Graduate Competency

Prepared Graduate Competencies are the P-12 concepts and skills that all students leaving the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Standards

Standards are the topical organization of an academic content area.

Grade Level Expectations

Expectations articulate, at each grade level, the knowledge and skills of a standard that indicates a student is making progress toward high school.

P-8

What do students need to know?

High School Expectations

High School

Expectations articulate the knowledge and skills of a standard that indicates a student is making progress toward being a prepared graduate.

What do students need to know?

Evidence Outcomes

Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.

How do we know that a student can do it?

21st Century and PWR Skills

Inquiry Questions:

Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

Relevance and Application:

Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

Nature of the Discipline:

The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Evidence Outcomes

Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.

How do we know that a student can do it?

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21st Century and PWR Skills

Inquiry Questions:

Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

Relevance and Application:

Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

Nature of the Discipline:

The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

STANDARDS TEMPLATE

Content Area: NAME OF CONTENT AREA

Standard: The topical organization of an academic content area.

Prepared Graduates:

> The P-12 concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting

High School and Grade Level Expectations

Concepts and skills students master:

Grade Level Expectation: High Schools: The articulation of the concepts and skills of a standard that indicates a student is making progress toward being a prepared graduate.

Grade Level Expectations: The articulation, at each grade level, the concepts and skills of a standard that indicates a student is making progress toward being ready for high school.

What do students need to know?

Evidence Outcomes	21 st Century Skills and Readiness Competencies
Students can:	Inquiry Questions:
Evidence outcomes are the indication that a student is meeting an expectation at the mastery level.	Sample questions intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.
	Relevance and Application:
How do we know that a student can do it?	Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.
	Nature of the Discipline:
	The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

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Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- ➤ Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- > Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- > Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- > Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

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Colorado Academic Standards Mathematics

The Colorado academic standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth-grade experience.

1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. Data

Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Modeling Across the Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards, specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

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Standards for Mathematical Practice from The Common Core State Standards for Mathematics

The Standards for Mathematical Practice have been included in the <u>Nature of Mathematics</u> section in each Grade Level Expectation of the Colorado Academic Standards. The following definitions and explanation of the Standards for Mathematical Practice from the Common Core State Standards can be found on pages 6, 7, and 8 in the Common Core State Standards for Mathematics. Each Mathematical Practices statement has been notated with (MP) at the end of the statement.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to <u>decontextualize</u>—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to <u>contextualize</u>, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. CDE: 5th Grade Mathematics

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Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or CDE: 5^{th} Grade Mathematics

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as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

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Mathematics Grade Level Expectations at a Glance

Standard	Grade Level	Expectation

	Grade Level Expectation
Fifth Grade	
1. Number Sense, Properties, and Operations	 The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency The concepts of multiplication and division can be applied to multiply and divide fractions
2. Patterns, Functions, and Algebraic Structures	1. Number patterns are based on operations and relationships
3. Data Analysis, Statistics, and Probability	1. Visual displays are used to interpret data
4. Shape, Dimension, and Geometric Relationships	 Properties of multiplication and addition provide the foundation for volume an attribute of solids Geometric figures can be described by their attributes and specific locations in the plane

From the Common State Standards for Mathematics, Page 33.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

- (1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
- (2) Students develop understanding of why division procedures work based on the meaning of baseten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

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(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

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21st Century Skills and Readiness Competencies in Mathematics

Mathematics in Colorado's description of $21^{\rm st}$ century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of Colorado $21^{\rm st}$ century skills, as follows:

Critical Thinking and Reasoning

Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

<u>Information Literacy</u>

The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

Collaboration

Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

Self-Direction

Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

<u>Invention</u>

Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

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Colorado's Description for School Readiness

(Adopted by the State Board of Education, December 2008)

School readiness describes both the preparedness of a child to engage in and benefit from learning experiences, and the ability of a school to meet the needs of all students enrolled in publicly funded preschools or kindergartens. School readiness is enhanced when schools, families, and community service providers work collaboratively to ensure that every child is ready for higher levels of learning in academic content.

Colorado's Description of Postsecondary and Workforce Readiness

(Adopted by the State Board of Education, June 2009)

Postsecondary and workforce readiness describes the knowledge, skills, and behaviors essential for high school graduates to be prepared to enter college and the workforce and to compete in the global economy. The description assumes students have developed consistent intellectual growth throughout their high school career as a result of academic work that is increasingly challenging, engaging, and coherent. Postsecondary education and workforce readiness assumes that students are ready and able to demonstrate the following without the need for remediation: Critical thinking and problem-solving; finding and using information/information technology; creativity and innovation; global and cultural awareness; civic responsibility; work ethic; personal responsibility; communication; and collaboration.

How These Skills and Competencies are Embedded in the Revised Standards

Three themes are used to describe these important skills and competencies and are interwoven throughout the standards: *inquiry questions; relevance and application; and the nature of each discipline.* These competencies should not be thought of stand-alone concepts, but should be integrated throughout the curriculum in all grade levels. Just as it is impossible to teach thinking skills to students without the content to think about, it is equally impossible for students to understand the content of a discipline without grappling with complex questions and the investigation of topics.

Inquiry Questions – Inquiry is a multifaceted process requiring students to think and pursue understanding. Inquiry demands that students (a) engage in an active observation and questioning process; (b) investigate to gather evidence; (c) formulate explanations based on evidence; (d) communicate and justify explanations, and; (e) reflect and refine ideas. Inquiry is more than hands-on activities; it requires students to cognitively wrestle with core concepts as they make sense of new ideas.

Relevance and Application – The hallmark of learning a discipline is the ability to apply the knowledge, skills, and concepts in real-world, relevant contexts. Components of this include solving problems, developing, adapting, and refining solutions for the betterment of society. The application of a discipline, including how technology assists or accelerates the work, enables students to more fully appreciate how the mastery of the grade level expectation matters after formal schooling is complete.

Nature of Discipline – The unique advantage of a discipline is the perspective it gives the mind to see the world and situations differently. The characteristics and viewpoint one keeps as a result of mastering the grade level expectation is the nature of the discipline retained in the mind's eye.

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1. Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- > Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- > Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- > Apply transformation to numbers, shapes, functional representations, and data

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Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

> Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. The decimal number system describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

Evidence Outcomes

Students can:

- Explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (CCSS: 5.NBT.1)
 - i. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. (CCSS: 5.NBT.2)
 - ii. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. (CCSS: 5.NBT.2)
 - iii. Use whole-number exponents to denote powers of 10. (CCSS: 5.NBT.2)
- b. Read, write, and compare decimals to thousandths. (CCSS: 5.NBT.3)
 - Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.¹ (CCSS: 5.NBT.3a)
 - ii. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (CCSS: 5.NBT.3b)
- c. Use place value understanding to round decimals to any place. (CCSS: 5.NBT.4)
- d. Convert like measurement units within a given measurement system. (CCSS: 5.MD)
 - i. Convert among different-sized standard measurement units within a given measurement system.² (CCSS: 5.MD.1)
 - ii. Use measurement conversions in solving multi-step, real world problems. (CCSS: 5.MD.1)

21st Century Skills and Readiness Competencies

Inquiry Questions:

- 1. What is the benefit of place value system?
- 2. What would it mean if we did not have a place value system?
- 3. What is the purpose of a place value system?
- 4. What is the purpose of zero in a place value system?

Relevance and Application:

1. Place value is applied to represent a myriad of numbers using only ten symbols.

Nature of Mathematics:

Revised: December 2010

- 1. Mathematicians use numbers like writers use letters to express ideas.
- 2. Mathematicians look closely and make use of structure by discerning patterns.
- 3. Mathematicians make sense of problems and persevere in solving them. (MP)
- 4. Mathematicians reason abstractly and quantitatively. (MP)
- 5. Mathematicians construct viable arguments and critique the reasoning of others. (MP)

Colorado Department of Education: 5th Grade Mathematics

Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency

Evidence Outcomes

Students can:

- a. Fluently multiply multi-digit whole numbers using standard algorithms. (CCSS: 5.NBT.5)
- b. Find whole-number quotients of whole numbers.³ (CCSS: 5.NBT.6)
 - i. Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 5.NBT.6)
 - ii. Illustrate and explain calculations by using equations, rectangular arrays, and/or area models. (CCSS: 5.NBT.6)
- c. Add, subtract, multiply, and divide decimals to hundredths. (CCSS: 5.NBT.7)
 - Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 5.NBT.7)
 - ii. Relate strategies to a written method and explain the reasoning used. (CCSS: 5.NBT.7)
- d. Write and interpret numerical expressions. (CCSS: 5.0A)
 - i. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (CCSS: 5.OA.1)
 - ii. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.⁴ (CCSS: 5.OA.2)

21st Century Skills and Readiness Competencies

Inquiry Questions:

- 1. How are mathematical operations related?
- 2. What makes one strategy or algorithm better than another?

Relevance and Application:

- 1. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra.
- 2. There are many models of multiplication and division such as the area model for tiling a floor and the repeated addition to group people for games.

Nature of Mathematics:

- 1. Mathematicians envision and test strategies for solving problems.
- 2. Mathematicians develop simple procedures to express complex mathematical concepts.
- 3. Mathematicians construct viable arguments and critique the reasoning of others. (MP)
- 4. Mathematicians model with mathematics. (MP)

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Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

> Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

3. Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency

Evidence Outcomes

Students can:

- a. Use equivalent fractions as a strategy to add and subtract fractions. (CCSS: 5.NF)
 - Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.⁵ (CCSS: 5.NF.2)
 - ii. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions⁶ with like denominators. (CCSS: 5.NF.1)
 - iii. Solve word problems involving addition and subtraction of fractions referring to the same whole. (CCSS: 5.NF.2)

21st Century Skills and Readiness Competencies

Inquiry Questions:

- 1. How do operations with fractions compare to operations with whole numbers?
- 2. Why are there more fractions than whole numbers?
- 3. Is there a smallest fraction?

Relevance and Application:

- 1. Computational fluency with fractions is necessary for activities in daily life such as cooking and measuring for household projects and crafts.
- 2. Estimation with fractions enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, the amount of carpeting needed for a room, or fencing required for a backyard.

Nature of Mathematics:

- 1. Mathematicians envision and test strategies for solving problems.
- 2. Mathematicians make sense of problems and persevere in solving them. (MP)
- 3. Mathematicians reason abstractly and quantitatively. (MP)
- 4. Mathematicians look for and make use of structure. (MP)

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Standard: 1. Number Sense, Properties, and Operations

Prepared Graduates:

> Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

4. The concepts of multiplication and division can be applied to multiply and divide fractions (CCSS: 5.NF)

Evidence Outcomes

Students can:

- a. Interpret a fraction as division of the numerator by the denominator $(a/b = a \div b)$. (CCSS: 5.NF.3)
- b. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.⁸ (CCSS: 5.NF.3)
- c. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. In general, $(a/b) \times (c/d) = ac/bd$. (CCSS: 5.NF.4a)
- d. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 5.NF.4b)
 - Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS: 5.NF.4b)
- e. Interpret multiplication as scaling (resizing). (CCSS: 5.NF.5)
 - i. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.¹⁰ (CCSS: 5.NF.5a)
 - ii. Apply the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. (CCSS: 5.NF.5b)
- f. Solve real world problems involving multiplication of fractions and mixed numbers. ¹¹ (CCSS: 5.NF.6)
- g. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. (CCSS: 5.NF.7a)
- h. Interpret division of a whole number by a unit fraction, and compute such quotients.¹³ (CCSS: 5.NF.7b)
- Solve real world problems involving division of unit fractions by nonzero whole numbers and division of whole numbers by unit fractions.¹⁴ (CCSS: 5.NF.7c)

21st Century Skills and Readiness Competencies

Inquiry Questions:

- Do adding and multiplying always result in an increase?
 Why?
- 2. Do subtracting and dividing always result in a decrease? Why?
- 3. How do operations with fractional numbers compare to operations with whole numbers?

Relevance and Application:

- 1. Rational numbers are used extensively in measurement tasks such as home remodeling, clothes alteration, graphic design, and engineering.
- 2. Situations from daily life can be modeled using operations with fractions, decimals, and percents such as determining the quantity of paint to buy or the number of pizzas to order for a large group.
- 3. Rational numbers are used to represent data and probability such as getting a certain color of gumball out of a machine, the probability that a batter will hit a home run, or the percent of a mountain covered in forest.

Nature of Mathematics:

Revised: December 2010

- 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.
- 2. Mathematicians make sense of problems and persevere in solving them. (MP)
- 3. Mathematicians model with mathematics. (MP)
- 4. Mathematicians look for and express regularity in repeated reasoning. (MP)

Colorado Department of Education: 5th Grade Mathematics

Standard: 1. Number Sense, Properties, and Operations Fifth Grade

Revised: December 2010

 $^{^{1}}$ e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$. (CCSS: 5.NBT.3a)

² e.g., convert 5 cm to 0.05 m. (CCSS: 5.MD.1)

³ with up to four-digit dividends and two-digit divisors. (CCSS: 5.NBT.6)

⁴ For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as 18932 + 921, without having to calculate the indicated sum or product. (CCSS: 5.OA.2)

⁵ For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2. (CCSS: 5.NF.2)

⁶ in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.). (CCSS: 5.NF.1)

⁷ including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.2)

⁸ e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? (CCSS: 5.NF.3)

 $^{^{9}}$ For example, use a visual fraction model to show (2/3) \times 4 = 8/3, and create a story context for this equation. Do the same with (2/3) \times (4/5) = 8/15. (CCSS: 5.NF.4a)

Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number. (CCSS: 5.NF.5b) Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number (CCSS: 5.NF.5b) 11 e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.6)

¹² For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$. (CCSS: 5.NF.7a)

¹³ For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$. (CCSS: 5.NF.7b)

¹⁴ e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins? (CCSS: 5.NF.7c)

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must have to ensure success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- > Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- > Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

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Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

> Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Number patterns are based on operations and relationships

1. Number patterns are based on operations a	na relationships
Evidence Outcomes	21 st Century Skills and Readiness Competencies
Students can:	Inquiry Questions:
a. Generate two numerical patterns using given rules. (CCSS:	1. How do you know when there is a pattern?
5.OA.3)	2. How are patterns useful?
b. Identify apparent relationships between corresponding terms.	
(CCSS: 5.OA.3)	
c. Form ordered pairs consisting of corresponding terms from the two	Relevance and Application:
patterns, and graphs the ordered pairs on a coordinate plane.	1. The use of a pattern of elapsed time helps to set up a
(CCSS: 5.OA.3)	schedule. For example, classes are each 50 minutes with 5
d. Explain informally relationships between corresponding terms in	minutes between each class.
the patterns. (CCSS: 5.0A.3)	2. The ability to use patterns allows problem-solving. For
e. Use patterns to solve problems including those involving saving	example, a rancher needs to know how many shoes to buy
and checking accounts ² (PFL)	for his horses, or a grocer needs to know how many cans
f. Explain, extend, and use patterns and relationships in solving	will fit on a set of shelves.
problems, including those involving saving and checking accounts such as understanding that spending more means saving less (PFL)	
Such as understanding that spending more means saving less (FFL)	Nature of Mathematics:
	 Mathematicians use creativity, invention, and ingenuity to understand and create patterns.
	2. The search for patterns can produce rewarding shortcuts
	and mathematical insights.
	3. Mathematicians construct viable arguments and critique the
	reasoning of others. (MP)
	4. Mathematicians model with mathematics. (MP)
	5. Mathematicians look for and express regularity in repeated
	reasoning. (MP)

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Standard: 2. Patterns, Functions, and Algebraic Structures Fifth Grade

¹ For example, given the rule "add 3" and the starting number 0, and given the rule "add 6" and the starting number 0, generate terms and the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. (CCSS: 5.OA.3)

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² such as the pattern created when saving \$10 a month

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- > Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

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Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:

> Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:			
1. Visual displays are used to interpret data			
Evidence Outcomes	21 st Century Skills and Readiness Competencies		
a. Represent and interpret data. (CCSS: 5.MD) i. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). (CCSS: 5.MD.2)	Inquiry Questions: 1. How can you make sense of the data you collect?		
ii. Use operations on fractions for this grade to solve problems involving information presented in line plots. ¹ (CCSS: 5.MD.2)	Relevance and Application: 1. The collection and analysis of data provides understanding of how things work. For example, measuring the temperature every day for a year helps to better understand weather.		
	Nature of Mathematics: 1. Mathematics helps people collect and use information to make good decisions. 2. Mathematicians model with mathematics. (MP) 3. Mathematicians use appropriate tools strategically. (MP) 4. Mathematicians attend to precision. (MP)		

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Standard: 3. Data Analysis, Statistics, and Probability Fifth Grade

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¹ For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. (CCSS: 5.MD.2)

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduates

The prepared graduate competencies are the preschool through twelfth-grade concepts and skills that all students who complete the Colorado education system must master to ensure their success in a postsecondary and workforce setting.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- > Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- > Apply transformation to numbers, shapes, functional representations, and data
- > Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- > Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

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Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

> Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Properties of multiplication and addition provide the foundation for volume an attribute of solids.

Evidence Outcomes	21 st Century Skills and Readiness Competencies
Students can:	Inquiry Questions:
 a. Model and justify the formula for volume of rectangular prisms. (CCSS: 5.MD.5b) i. Model the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes.¹ (CCSS: 5.MD.5b) 	Why do you think a unit cube is used to measure volume?
 ii. Show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. (CCSS: 5.MD.5a) iii. Represent threefold whole-number products as volumes to represent the associative property of multiplication. (CCSS: 5.MD.5a) b. Find volume of rectangular prisms using a variety of methods and use these techniques to solve real world and mathematical 	Relevance and Application: 1. The ability to find volume helps to answer important questions such as which container holds more.
 i. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (CCSS: 5.MD.4) ii. Apply the formulas V = I × w × h and V = b × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths. (CCSS: 5.MD.5b) iii. Use the additive nature of volume to find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts. (CCSS: 5.MD.5c) 	 Nature of Mathematics: Mathematicians create visual and physical representations of problems and ideas that reveal relationships and meaning. Mathematicians make sense of problems and persevere in solving them. (MP) Mathematicians model with mathematics. (MP)

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Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

> Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. Geometric figures can be described by their attributes and specific locations in the plane

Evidence Outcomes

Students can:

- a. Graph points on the coordinate plane² to solve realworld and mathematical problems. (CCSS: 5.G)
- b. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (CCSS: 5.G.2)
- c. Classify two-dimensional figures into categories based on their properties. (CCSS: 5.G)
 - i. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.³ (CCSS: 5.G.3)
 - ii. Classify two-dimensional figures in a hierarchy based on properties. (CCSS: 5.G.4)

21st Century Skills and Readiness Competencies

Inquiry Questions:

- 1. How does using a coordinate grid help us solve real world problems?
- 1. What are the ways to compare and classify geometric figures?
- 2. Why do we classify shapes?

Relevance and Application:

- 1. The coordinate grid is a basic example of a system for mapping relative locations of objects. It provides a basis for understanding latitude and longitude, GPS coordinates, and all kinds of geographic maps.
- Symmetry is used to analyze features of complex systems and to create worlds of art. For example symmetry is found in living organisms, the art of MC Escher, and the design of tile patterns, and wallpaper.

Nature of Mathematics:

- 1. Geometry's attributes give the mind the right tools to consider the world around us.
- 2. Mathematicians model with mathematics. (MP)
- 3. Mathematicians look for and make use of structure. (MP)

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Standard: 4. Shape, Dimension, and Geometric Relationships Fifth Grade

A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. (CCSS: 5.MD.3b) ² Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. (CCSS: 5.G.1)

Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). (CCSS: 5.G.1)

³ For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. (CCSS: 5.G.3)

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¹ A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. (CCSS: 5.MD.3a)