Sample Performance Assessment

Content Area: Mathematics

Grade Level: High School, Algebra I

Instructional Unit Sample: Power to the Variable

Colorado Academic Standard(s): MA10-GR.HS-S.1-GLE.2; MA10-GR.HS-S.2-GLE.2; MA10-GR.HS-S.2-GLE.3; MA10-GR.HS-S.2-GLE.4

Concepts and skills students' master:

- Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively.
- Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (MA10-GR.HS-S.1-GLE.2-EOa.i.1,2)
- Create equations and inequalities in one variable and use them to solve problems; include equations arising from linear, quadratic, and exponential function with integer exponents. (MA10-GR.HS-S.2-GLE.4-EO.a.i)
- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (MA10-GR.HS-S.2-GLE.2-EO.a.i.2)
- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (MA10-GR.HS-S.2-GLE.2-EO.a.i.3)
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (MA10-GR.HS-S.2-GLE.2-EO.a.ii)
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (MA10-GR.HS-S.2-GLE.2-EO.a.iii)
- Interpret the parameters in a linear or exponential (domain of integers) function in terms of a real world context and prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (MA10-GR.HS-S.2-GLE.2-EO.b.i)

Unit Description

This unit, Power to the Variable, focuses on a formal introduction to exponential functions. The students start with exploring exponential growth through geometric sequences that either grow or decay. As the students learn about geometric sequences, they continually compare them to arithmetic sequences, building to linear and exponential functions. Student fluency with these functions improves through multiple experiences with tables, graphs, equations and contexts. Then students examine the differences in the growth rates of linear, exponential, and polynomial functions leading to a formal proof of how linear functions grow by constant differences and exponential functions grow by common factors.

COLORADO Department of Education

Performance Assessment Description

You are a scientist who works at Mauna Loa observatory in Hawaii who measures CO_2 concentration in the atmosphere. You have data from the last 50 years (http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_full). You are presenting to the governor about your data, including a prediction about what the CO_2 level will be in Hawaii in the year 2050. In order to create your prediction you will need to determine if it should be modeled by a linear or exponential function based on the rate of growth.

[Note: A teacher may choose to pick another set of data to work with if it will be more engaging or relevant to their students]

RUBRIC: Power to the Variable

	Above Mastery	Mastery of Grade Level Standards	Approaching Mastery	Novice	
<u>Scoring Criteria</u>	4	3	2	1	Weight
Modeling **Devises and enacts a plan to apply mathematics in solving problems arising in everyday life, society and the workplace by:**	Using stated assumptions **and making assumptions** and approximations to simplify a real-world situation **(includes micro-models)** *(Expands beyond the linear/exponential micro-model to address other models.)* **Mapping relationships between** important quantities *(Compares models other than their chosen equation output to given data.)* **Selecting appropriate** tools to create models *(Uses a statistics program.)* Analyzing relationships mathematically between important quantities to draw conclusions *(Draws a bounded conclusion based on their model that addresses limitations.)*	Using stated assumptions **and making assumptions** and approximations to simplify a real-world situation **(includes micro-models)** *(Uses all data from the web site to establish micro-model and strategically narrows down what data to use.)* **Mapping relationships between** important quantities *(Compares their equation output to given data.)* **Selecting appropriate** tools to create models *(Uses the data provided on the web site and a spread sheet.)* Analyzing relationships mathematically between important quantities to draw conclusions *(Draws conclusion based on their model that addresses limitations eg. Extrapolation.)* Interpreting mathematical **results in the context of the situation** *(Interprets results based on context.)* Reflecting on whether the	Using stated assumptions and approximations to simplify a real-world situation *(Uses all data from the web site without distinction.)* **Illustrating** relationships between important quantities (Graphs an equation which models *time vs CO2 levels.)* Using provided tools to create models *(Uses a graphing calculator.)* Analyzing relationships mathematically **between important quantities** to draw conclusions *(Draws conclusion based on their model.)* **Interpreting mathematical results in a simplified context** *(Interprets based on their model.)* **Reflecting on whether the**	Using stated assumptions and approximations to simplify a real-world situation *(Uses partial data from the web site, eg: read points from the graph)* Identifying important quantities *(Identifies time vs CO2 levels)* Using provided tools to create models *(Uses graph provided on the web site.)* Analyzing relationships mathematically to draw conclusions *(Draws a conclusion.)* No evidence of interpreting results No evidence of reflection	X 1

		results make sense *(Reflection references context.)* **Improving** the model if it has not served its purpose *(Created two models, compares a linear versus an exponential mode, and justifies their choice.)* Writing a **complete, clear and correct** algebraic expression or equation to describe a situation *(Must include a definition of all variables.)*	**results make sense** *(Reflection does not include context.)* **Modifying the model if it has not served its purpose** *(Created two models and chooses one without explanation.)* Writing an algebraic expression or equation to describe a situation *(States the parts of the equation but not the equation, eg calculator output.)*	 No evidence of modification to their model. *(Chooses one model without explanation.)* Writing an algebraic expression or equation to describe a situation *(Creates a linear equation based on two points.)*	
Content	Represents linear and exponential (with domain in the integers) functions symbolically, in real-life scenarios, graphically, with a verbal description, as a sequence and with input-output pairs to solve mathematical and contextual problems. *(Provides in context an estimate for 2050 based on the model chosen **and** assesses reasonableness in comparison to other models beyond linear and exponential.)*	Represents linear and exponential (with domain in the integers) functions symbolically, **in real-life scenarios**, graphically, with a verbal description, as a sequence and with input-output pairs to solve mathematical **and contextual problems**. *(Provides, in context, an estimate for 2050 based on the model chosen and assesses reasonableness.)*	**Represents** linear and exponential (with domain in the integers) functions symbolically, graphically and with input-output pairs to solve mathematical problems. *(Provides an estimate for 2050 based on the model chosen.)*	Given a symbolic representation, real-life scenario, graph, verbal description, sequence or input-output pairs for linear and exponential functions (with domains in the integers), solves mathematical problems. *(Provides an estimate for 2050 without explanation.)*	X 1
Communication	Provides effective development of the evidence necessary to support their reasoning and answer **and compares/contrasts their conclusion versus other models.**	Provides **effective** development of the evidence necessary to support their reasoning and answer. *(This development may be a linear or exponential model; builds a convincing argument to support their choice of models. Must include a*	Provides **some** development of the evidence necessary to support their reasoning and answer. *(The work is shown, reference to assumptions are made, and a model is chosen with limited reasoning supporting their choice.)*	Provides **minimal** development of the evidence necessary to support their reasoning and answer.	X 1

		definition of all variables.) In writing, the student demonstrates **effective** command of the conventions of mathematical vocabulary. (*chooses appropriate academic and technical vocabulary)*	In writing, the student demonstrates **limited** command of the conventions of mathematical vocabulary. (*chooses informal vocabulary)*	In writing, the student demonstrates **minimal** command of the conventions of mathematical vocabulary.	
				TOTAL	

Performance Assessment Development Template

Who is developing this performance assessment?	
Name: Colorado Content Collaborative in Mathematics	**Position/Affiliation:** Colorado Content Collaborative in Mathematics

I. CONTENT STANDARDS	
Content Area: Mathematics	
Colorado Academic Standards Specify the Colorado Academic Standard(s) that will be evaluated by the performance tasks. Colorado Academic Standards Online (hold CTRL and click to visit the website)	➢ Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (MA10-GR.HS-S.1-GLE.2-EOa.i.1,2) ➢ Create equations and inequalities in one variable and use them to solve problems; include equations arising from linear, quadratic, and exponential function with integer exponents. (MA10-GR.HS-S.2-GLE.4-EO.a.i) ➢ Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (MA10-GR.HS-S.2-GLE.2-EO.a.i.2) ➢ Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (MA10-GR.HS-S.2-GLE.2-EO.a.i.3) ➢ Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (MA10-GR.HS-S.2-GLE.2-EO.a.ii) ➢ Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (MA10-GR.HS-S.2-GLE.2-EO.a.iii) ➢ Interpret the parameters in a linear or exponential (domain of integers) function in terms of a real world context and prove that linear functions grow by equal differences over

	equal intervals, and that exponential functions grow by equal factors over equal intervals. (MA10-GR.HS-S.2-GLE.2-EO.b.i)
Grade Level(s)	**High School, Algebra 1**
Indicate the intended Depth of Knowledge (DOK) for this performance assessment.	☐**DOK 1** ☐**DOK 2** ☐**DOK 3** ☑**DOK 4**
What are some real-world situations that relate to the content standards above? Some examples are included in the Colorado standards under "Relevance and Application."	➢ Population ➢ Debt ➢ Savings ➢ Radioactivity

Summary. Provide a brief summary describing the task in the boxes below.	
Performance Task Name **Power to the Variable -- Mauna Loa Observatory**	**Brief Description of the Task** You are a scientist who works at Mauna Loa observatory in Hawaii who measures CO2 concentration in the atmosphere. You have data from the last 50 years (http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_full). You are presenting to the governor about your data, including a prediction about what the CO2 level will be in Hawaii in the year 2050. In order to create your prediction you will need to determine if it should be modeled by a linear or exponential function based on the rate of growth. [Note: A teacher may choose to pick another set of data to work with if it will be more engaging or relevant to their students]

II. Claims, Skills, Knowledge & Evidence	
Claims. *What claim(s) do you wish to make about the student? In other words, what inferences do you wish to make about what a student knows or can do? Define any key concepts in these claims.*	**Successful completion of this task would indicate…** ➢ Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively.
Skills. *Refer to the standard(s), grade level, and DOK levels you listed in Section I. Given this information, what skills should be*	**Student should be able to…** ➢ Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units

assessed? All skills should align with the above claims.	consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (MA10-GR.HS-S.1-GLE.2-EOa.i.1,2) ➢ Create equations and inequalities in one variable and use them to solve problems; include equations arising from linear, quadratic, and exponential function with integer exponents. (MA10-GR.HS-S.2-GLE.4-EO.a.i) ➢ Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (MA10-GR.HS-S.2-GLE.2-EO.a.i.2) ➢ Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (MA10-GR.HS-S.2-GLE.2-EO.a.i.3) ➢ Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (MA10-GR.HS-S.2-GLE.2-EO.a.ii) ➢ Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (MA10-GR.HS-S.2-GLE.2-EO.a.iii) ➢ Interpret the parameters in a linear or exponential (domain of integers) function in terms of a real world context and prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (MA10-GR.HS-S.2-GLE.2-EO.b.i)
Knowledge. *Refer to the standard(s), grade level, and DOK level you listed in Section I. Given this information, what knowledge/concepts should be assessed? All knowledge should align with the above claims.*	**Student should know/understand…** ➢ Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively.
Evidence. *What can the student do/produce to show evidence of the above knowledge and skills?*	**Student will show evidence of skills and knowledge by…** ➢ Provide a written report of each part of the modeling process described above and a statement of final conclusions including the limitations of each model. ➢ Create a 2 to 3 minute digital story (video) that the governor would be able to watch and understand the prediction and the limitations of the prediction.

III.A. PERFORMANCE TASKS: Instructions to the Student
Think about the performance assessment process from a student's perspective. What instructions does the student need? Make sure the instructions are <u>fair and unbiased</u>. Instructions should be detailed, clear, and written at the appropriate grade level. You are a scientist who works at Mauna Loa observatory in Hawaii who measures CO2 concentration in the atmosphere. You have data from the last 50 years (http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_full). You are presenting to the governor about your data, including a prediction about what the CO2 level will be in Hawaii in the year 2050. In order to create your prediction you will need to determine if it should be modeled by a linear or exponential function based on the rate of growth. [Note: A teacher may choose to pick another set of data to work with if it will be more engaging or relevant to their students]
Give the student an overview of the performance assessment (i.e., purpose of the assessment, tasks the student will need to complete, etc.). You are a scientist who works at Mauna Loa observatory in Hawaii who measures CO2 concentration in the atmosphere. You have data from the last 50 years (http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_full). You are presenting to the governor about your data, including a prediction about what the CO2 level will be in Hawaii in the year 2050. In order to create your prediction you will need to determine if it should be modeled by a linear or exponential function based on the rate of growth.
Stimulus Material. ***Describe what stimulus material the student will receive. For example, the stimulus might be a story or scenario that the student reads, analyzes, and to which the student provides a response.*** You are a scientist who works at Mauna Loa observatory in Hawaii who measures CO2 concentration in the atmosphere. You have data from the last 50 years (http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_full). You are presenting to the governor about your data, including a prediction about what the CO2 level will be in Hawaii in the year 2050. In order to create your prediction you will need to determine if it should be modeled by a linear or exponential function based on the rate of growth.
Explain to the student what documents/materials they have for the performance assessment. Explain what the student should <u>do</u> with those documents/materials. (http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_full).

Students will model the data with both a linear function and an exponential function and use it to predict the CO2 concentration in the year 2050 and also when the CO2 concentration will be above 500 parts per million. Students will discuss and document which model makes a better prediction and the limitations of each model.

Students will be expected to show evidence of each step of the modeling process:

- Problem – Show an understanding of what is being asked and what they are modeling.
- Formulate – Create a linear and exponential function for the data showing each model in an equation, graph and table.
- Compute – Calculate the CO2 levels for 2050 and the date at which the CO2 will be above 500 parts per million with each model.
- Interpret – Interpret the parameters of the functions in the context of CO2 concentration over time.
- Validate – Check the model for accuracy by predicting the CO2 concentration for intermediate years to assess the reasonableness of each model.

Students will have the following products for their report:

- Provide a written report of each part of the modeling process described above and a statement of final conclusions including the limitations of each model.
- Create a 2 to 3 minute digital story (video) that the governor would be able to watch and understand the prediction and the limitations of the prediction.

Describe in detail any safety equipment that is required. Is safety equipment provided onsite, or are students expected to bring their own safety equipment?

n/a

Explain what students need to do when they complete each task (e.g., submit work to the educator, move on to the next task, etc.).

n/a

Provide any other relevant information for the students' instructions.

n/a

III.B. PERFORMANCE TASKS: Instructions to the Educator
Think about the performance assessment process from an educator's perspective. What instructions do educators need? Instructions to the educator should be clear and concise.
Before the Performance Assessment is Administered ***How should the educator prepare the site where the performance assessment will be administered? Be as specific as possible.*** Students need access to the web site for the data. ***What materials should be provided to students? Be as specific as possible.*** n/a ***What materials should the student bring to the performance assessment session? Be as specific as possible.*** n/a ***What materials should <u>not</u> be available to the student during the performance assessment session (e.g., cell phones, calculators, etc.)?*** n/a
Should the educator keep track of time? If so, specify how much time the student will have to complete the performance assessment. Explain how the educator should keep track of and record time. n/a
Will the educator need to video/audio record the students during the performance assessment session? If so, provide detailed instructions on how to set up the recording equipment. n/a

During the Performance Assessment Session

How should the educator respond to students' questions?

Educators should respond to clarifying questions asked by the students.

What should the educator do while the student is completing the tasks (e.g., should the educator make notes about the student's process, mark scores on rubrics, etc.)?

n/a

Upon Completion of the Performance Assessment

What does the educator need to collect from the student?

The written report and the digital video should be collected.

What information should the educator give the student at the end of the performance assessment session?

n/a

Who is responsible for cleaning/resetting the workstation (if necessary)—the student or the educator? How should the workstation be cleaned?

n/a

Other relevant information for the *educator*'s instructions:

n/a

III.C. PERFORMANCE TASKS: Other Considerations

How will students' responses be recorded? ***Describe how evidence will be collected about each student's performance (e.g., student submits a work product, educator records information about the student's process, etc.)***

Students will complete a report and digital story.

What needs to be built for this performance assessment? *Refer to the materials list above. Think about what materials must be created for this performance assessment. Some examples include: worksheets, instruction sheets for the educator, videos, websites, etc.* For differentiation, students can write their report using a template showing each step of the modeling process and sentence starters to scaffold their writing. (http://www.mathsisfun.com/algebra/mathematical-models.html, http://caccssm.cmpso.org/high-school-modeling-task-force)

III.D. PERFORMANCE TASKS: Accommodations
What are the requirements for this set of tasks? What accommodations might be needed? List all accommodations that might apply (e.g., accommodations for language, timing, setting, etc.). For differentiation, students can write their report using a template showing each step of the modeling process and sentence starters to scaffold their writing. (http://www.mathsisfun.com/algebra/mathematical-models.html, http://caccssm.cmpso.org/high-school-modeling-task-force)

IV. EDUCATOR INFORMATION
What are the requirements to be an educator for this performance assessment? What are the knowledge and skills and educator must possess in order to successfully administer and score this performance assessment. Please provide your recommendations below. The educator needs to be a secondary mathematics teacher.

Performance Assessment Development Process

The work of the Colorado Content Collaboratives is intended to support effective instructional practice by providing high quality examples of assessment and how assessment information is used to promote student learning.

The new Colorado Academic Standards require students to apply content knowledge using extended conceptual thinking and 21st century skills. Performance assessments have the highest capacity to not only measure student mastery of the standards but also provide the most instructionally relevant information to educators. Further, performance assessments can integrate multiple standards within and across content areas, providing educators a comprehensive perspective of student knowledge and giving students the opportunity to demonstrate the degree to which they understand and transfer their knowledge.

Performance Assessment - An assessment based on observation and judgment. It has two parts: the task and the criteria for judging quality. Students complete a task (give a demonstration or create a product) and it is evaluated by judging the level of quality using a rubric. Examples of demonstrations include playing a musical instrument, carrying out the steps in a scientific experiment, speaking a foreign language, reading aloud with fluency, repairing an engine, or working productively in a group. Examples of products can include writing an essay, producing a work of art, writing a lab report, etc. (Pearson Training Institute, 2011)

The Content Collaboratives worked closely with the Center for Educational Testing and Evaluation from the University of Kansas to establish protocols for the development of performance assessments and to use those protocols to develop performance assessments that include scoring rubrics. The Performance Assessment Development Process includes a collection of resources to aid schools and districts that choose to engage in locally developing performance assessments. These resources can be accessed in the CDE Assessment Resource Bank at http://www.coloradoplc.org/node/12765.

The Performance Assessment Development Process is best utilized when intending to create an assessment for culminating assessment purposes such as a unit, end of course, end of semester, or end of year summative assessment. Additionally, a district, BOCES, or school may wish to create a common performance assessment that can be used across multiple classrooms. Engaging in the Performance Assessment Development Process serves as evidence that an educator is participating in valuable assessment work that aligns to the Colorado Academic Standards, district curriculum, and district goals.

The performance assessments developed by the Content Collaboratives serve as high-quality examples of performance assessments that can be used for a variety of purposes. Scores from these performance assessments are used at the discretion of the district or school.