Balanced Assessment

HIGH SCHOOL ASSESSMENT

PACKAGE 2

Balanced Assessment for the Mathematics Curriculum

BERKELEY ■ HARVARD ■ MICHIGAN STATE ■ SHELL CENTRE

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Expanded Table of Contents*

Long Tasks	Task Type	Circumstances of Performanc
1. Chocolate Polyhedra	45-minute pure investigation, with some nonroutine aspects of mathematics in a nonroutine context; open-ended	individual written response
2. Ordering a Cab	45-minute recommendation task; applied power in a nonroutine context from student life	individual written response after discussion in pairs
3. Sort Them	45-minute problem in pure mathematics; nonroutine approach	individual written response after discussion in pairs
4. House in a Hurry	45-minute planning; applied power in a nonroutine context from adult life; open-ended	individual written response after discussion in pairs
5. Checking an Odometer	45-minute problem; illustrative application of proportional reasoning in a nonroutine context from student life	individual written response
6. Designing a Tent	45-minute design task; applied power in a nonroutine context from student life	individual written response after a discussion in pairs
7. 2000% Blowup	45-minute problem; illustrative application of proportional reasoning in a nonroutine context from student life	individual written response after a discussion in pairs
8. Cross the Box	60-minute open investigation; applied power in a nonroutine context from student life; open-ended	individual written response after a discussion in pairs

* For explanations of terms that may be unfamiliar, see the Glossary, and the *Dimensions of Balance* table in the Introductio

Mathematical Content	Mathematical Processes
Geometry, Shape, and Space: properties of sections of a cube by various mid-planes; strong visualization demand; investigation of Euler's formula for polyhedra, in this context and beyond	formulation with interpretation and evaluation of the results important for checking
Data, Statistics, and Probability: choice and use of appropriate representations of data for analysis of the response times of cabs; construction of competing arguments	representation, interpretation, and evaluation of the data; formulation and communication of the arguments
Patterns, Functions, and Algebra: sorting and connecting the tabular, algebraic, graphical, and verbal representations of 10 simple functions	interpretation of the given representations, based on understanding of the transformations between them
Other Mathematics: the discrete mathematics of scheduling jobs, successively and in parallel, is approached informally; student must devise and use appropriate charts and diagrams	formulation of a systematic approach to the problem; manipulation of the given data; interpretation, evaluation, and communication of the results
Patterns, Functions, and Algebra: recognizing the need for, and using proportional reasoning in relating an odometer which reads 15% low to the real distances; forward and reverse reasoning	formulation of a model; transformation of the data
Geometry, Shape, and Space, with Number: estimation of sizes of people and tent dimensions; visualizing shape of a net; Pythagorean theorem and/or trigonometry for lengths and angles	formulation of the estimates and the net shape; manipulations for calculating the lengths and angles
Geometry, Shape, and Space, with Number: proportional reasoning in a geometric situation involving measurements on two enlarged photos, with inferences about the negative	formulation of the approach; manipulations, both measurement and computation, inference about the negative
Data, Statistics, and Probability: probability distribution of the difference of two dice; collecting and analyzing data; inferring best strategy	manipulation in collecting and analyzing data; inference and formulation of a strategy

Expanded Table of Contents

Short Tasks	Task Type	Circumstances of Performan
9. The Magazine	15-minute task; applied power with open paths to nonroutine math connections in an adult-life context	individual written response
10. Kitchen Tiles	15-minute nonroutine problem in pure mathematics; almost an exercise	individual written response
11. Make it Bigger	15-minute problem, nonroutine in math results and in context; applied power	individual written response discussion in pairs
12. House Plan	15-minute exercise; applied power in a nonroutine context	individual written response

High School Package 2

Mathematical Content	Mathematical Processes
Algebra and Number, with functional relationships in numerical and graphical form	mainly manipulation, with some interpretation
Algebra and Function in a pure geometric context; recognizing patterns and representing them in graphs and symbols	mainly manipulation, with some evaluation of results
Functional relationships for scaling; involves number in an everyday geometric context	mainly manipulation but with a significant formulation aspect
Measurement and computation to relate a scale drawing to the actual house	mainly manipulation, with some interpretation

Name Date

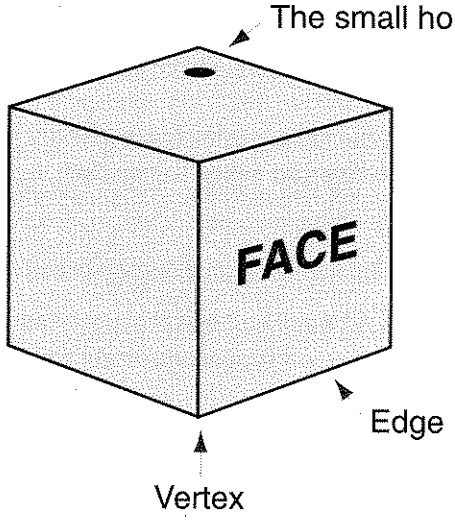
Chocolate Polyhedra

This problem gives you the chance to

- *show how you can visualize 3-dimensional shapes*
- *carry out an investigation*

Imagine that you work in a chocolate factory and that you are responsible for designing chocolates into interesting shapes.

You have several plastic molds in the shape of a cube. They look like the one shown here.



Melted chocolate is poured into each mold through the small hole so that when set, the mold is exactly half full.

To make chocolates in different shapes, place the molds in different positions to set the chocolate.

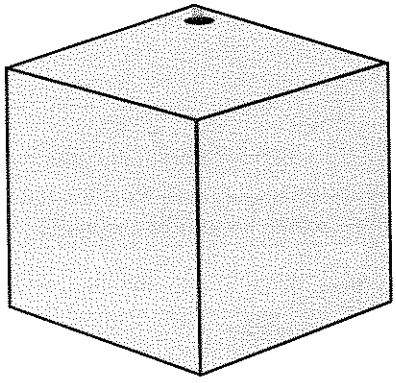
Each of the diagrams on the next two pages show the position in which the mold was left to set.

Look at each of the diagrams.

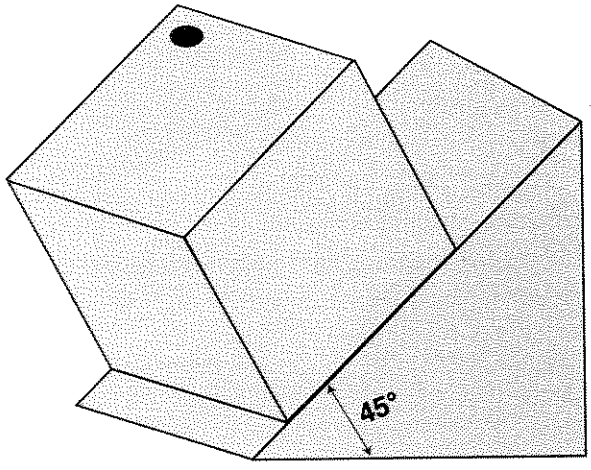
- Make a sketch of the chocolate piece that is made.
- Record the number of faces.
- Record the number of corners (vertices).
- Record the number of edges.
- Describe, as fully as possible, the shape that is made.

Name Date

1. The mold sets while resting on one face.



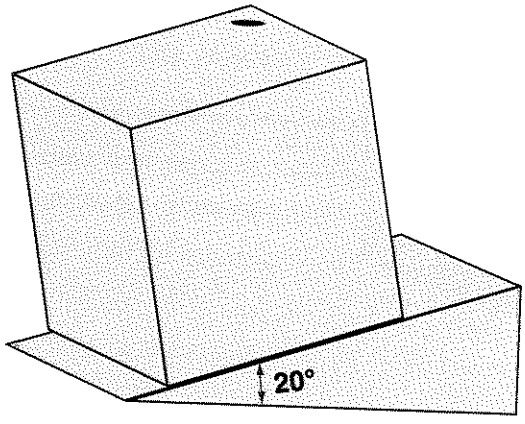
2. The mold sets while balanced perfectly on one edge at an angle of 45° to the horizontal.



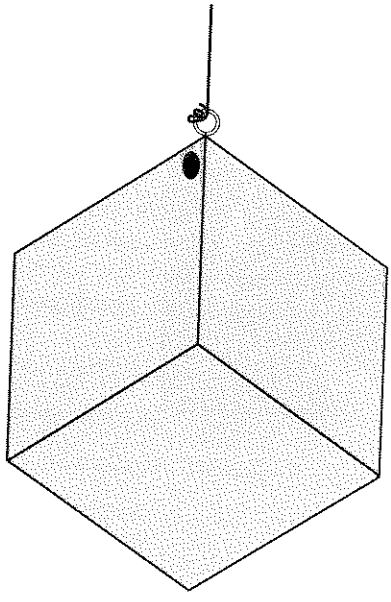
Name

Date

3. The mold sets while tilted on one edge at an angle of 20° to the horizontal.



4. The mold sets while balanced perfectly on one vertex.



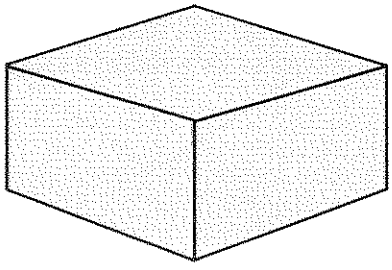
A Sample Solution

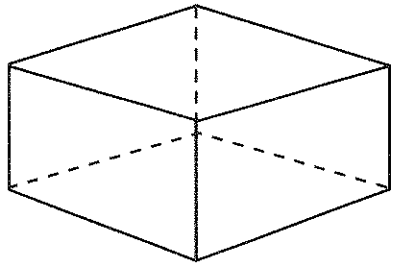
Task 1

See below for sketches and descriptions.

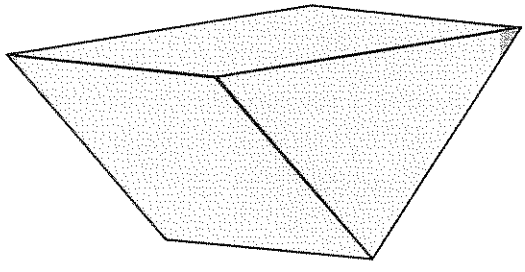
Position	Number of faces	Number of corners	Number of edges
1. Resting on a face	6	8	12
2. Resting on an edge (45°)	5	6	9
3. Resting on an edge (20°)	6	8	12
4. Resting on one vertex	7	10	15

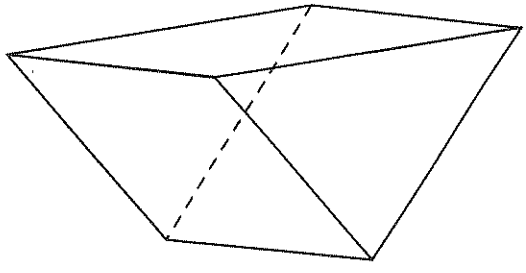
1. The shape is a rectangular solid.





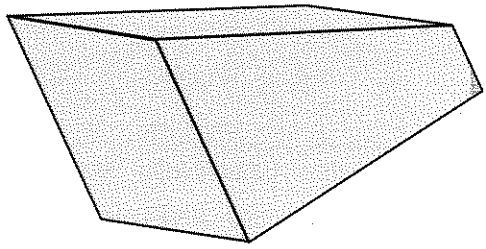
2. The shape is a triangular right prism in which the bases are right triangles.

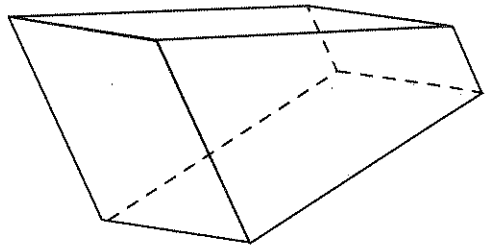




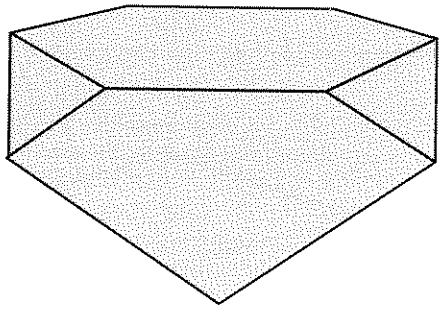
Task 1

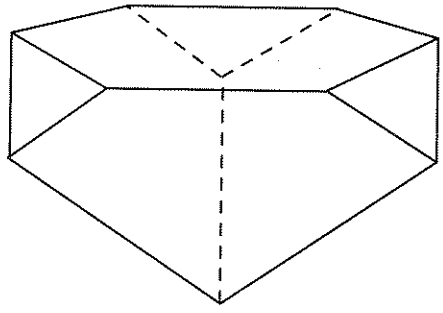
3. The shape is a trapezoidal right prism in which the bases are right trapezoids.





4. The shape is a triply truncated tetrahedron. The three faces of truncation are congruent equilateral triangles.





Using this Task

Task 1

Extensions

Euler's Formula says that for any polyhedron:

> the number of faces (F) + the number of vertices (V) – the number of edges (E) is equal to a certain number.

Devise and report an investigation of this formula. Find out what that certain number is. You may use this work or any example of polyhedra. Packaging, especially packaging used for chocolate boxes, is a good source. In your report include interesting polyhedra that illustrate Euler's Formula.

For Formal Assessment

Students usually find question 4 challenging. A clear cube that is half full of salt may be necessary to enable students to see the correct solution.

Task 1

Characterizing Performance

This section offers a characterization of student responses and provides indications of the ways in which the students were successful or unsuccessful in engaging with and completing the task. The descriptions are keyed to the *Core Elements of Performance.* Our global descriptions of student work range from "The student needs significant instruction" to "The student's work meets the essential demands of the task." Samples of student work that exemplify these descriptions of performance are included below, accompanied by commentary on central aspects of each student's response. These sample responses are *representative;* they may not mirror the global description of performance in all respects, being weaker in some and stronger in others.

The characterization of student responses for this task is based on these *Core Elements of Performance:*

1. Visualize 3-D shapes.
2. Sketch polyhedra.
3. Use Euler's Formula, $F + V - E$ to find a certain number.

Descriptions of Student Work

The student needs significant instruction.

These papers show, at most, evidence of clear understanding of what the task is asking. Typically the student might attempt the first part but might do so with little success.

Student A

This response shows that the student has attempted to engage with the task but has found it difficult to visualize and/or communicate the most straightforward situation.

Task

The student needs some instruction.

These papers provide evidence that the student can visualize and communicate the relatively straightforward situations.

Typically the response will provide the correct solution for the first and second orientation.

Student B

This response shows that the student can visualize and communicate 3-dimensional shapes only in relatively uncomplicated situations.

The student's work needs to be revised.

The student will have completed the first three orientations correctly, and completed Euler's formula.

Student C

This response shows that the student can visualize and communicate 3-dimensional shapes. It is expected that in a revision of this paper the student will complete Euler's formula and devise a way of visualizing the fourth orientation.

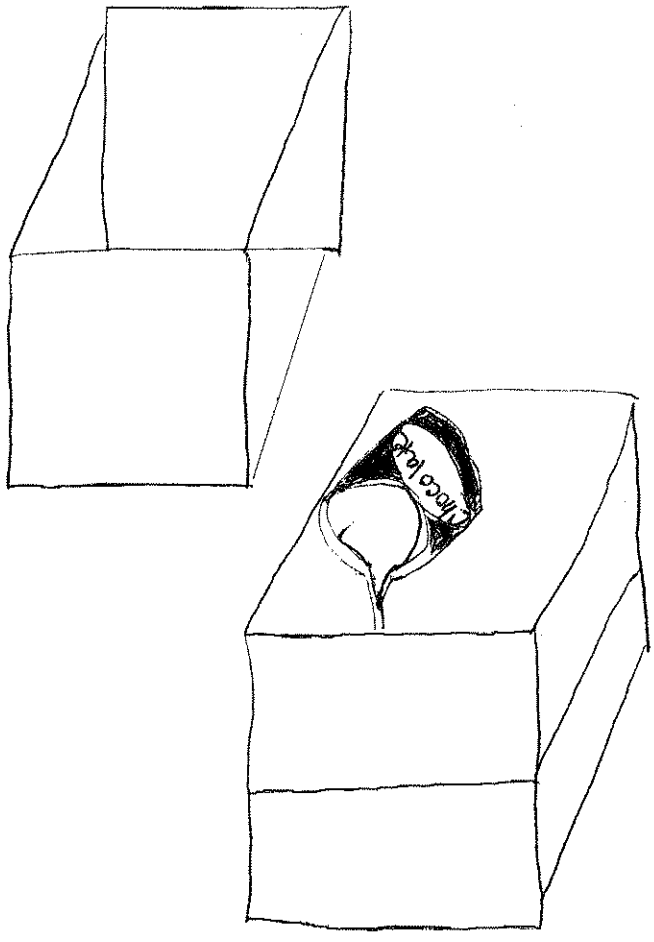
The student's work meets the essential demands of the task.

The fourth orientation is exceptionally difficult and it is expected that few students will give the correct solution in an on-demand setting. Therefore, a response could *meet the essential demands of the task* without providing a correct solution to the fourth orientation. It is expected that all other aspects of the response will be correct.

Student D

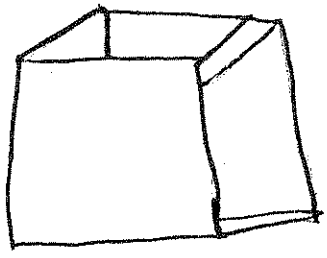
This response shows that the student can visualize and communicate 3-dimensional shapes. With the exception of the fourth orientation, all aspects of the response are correct.

Student A

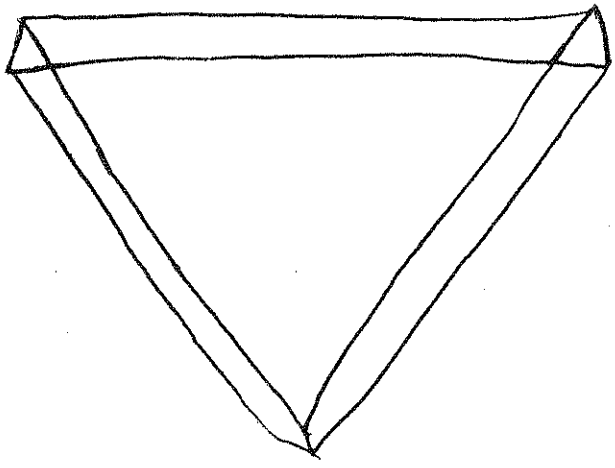


Faces - 6
Edge - 8
Vertex - 4

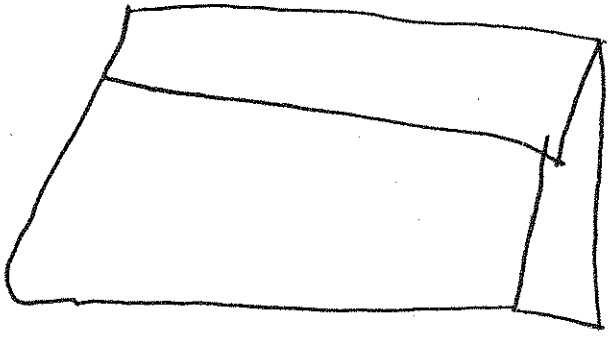
Student B



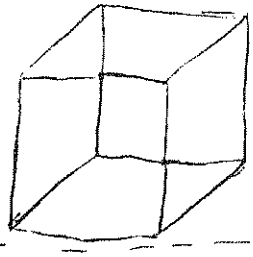
faces = 6
corners = 8
edges = 12



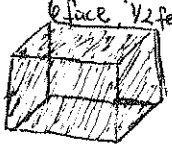
5 faces
6 corners
9 edges



Student C

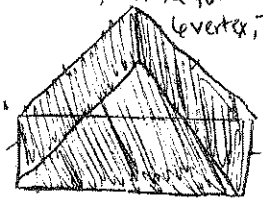


8 vertex, 12 edges
6 face, 1/2 feet



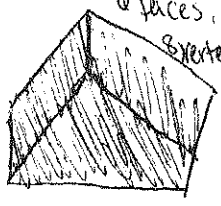
2.

5 face, 1/2 full
6 vertex, 7 edges



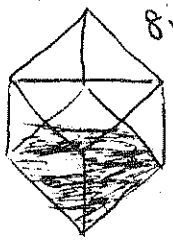
3.

6 faces, 1/2 full
8 vertex, 12 edges



6 faces, 12 edges, 1/2 full
8 vertex

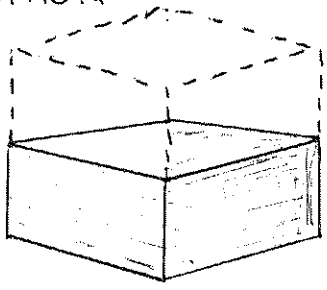
1.



we got a lot of 8 vertex
12 edges

Student D

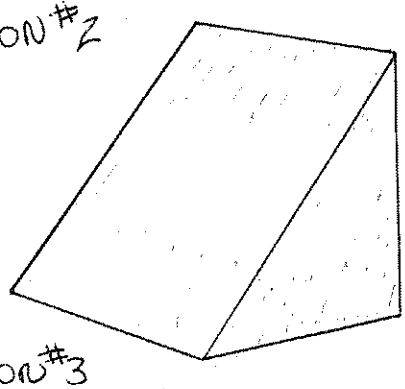
Position #1



OF FACES = 6
OF CORNERS = 8
OF EDGES = 12

14 + 12 = 2

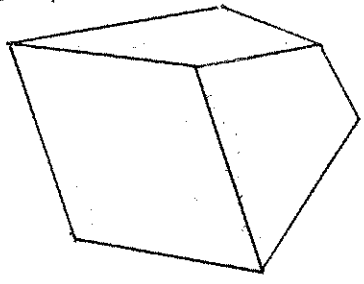
Position #2



OF FACES = 5
OF CORNERS = 6
OF EDGES = 9

11 - 9 = 2

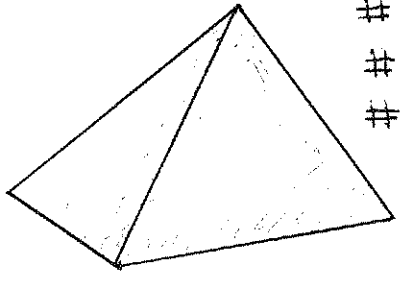
Position #3



OF FACES = 6
CORNERS = 8
EDGES = 12

14 - 12 = 2

position #4



OF FACES = 5
OF CORNERS = 5
OF EDGES = 8

10 - 8 = 2