Balanced Assessment

HIGH SCHOOL ASSESSMENT

PACKAGE 1

Balanced Assessment for the Mathematics Curriculum

BERKELEY ▪ HARVARD ▪ MICHIGAN STATE ▪ SHELL CENTRE

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Expanded Table of Contents*

Long Tasks	Task Type	Circumstances of Performance
1. **Strange but True**	45-minute problem illustrating an application of the mathematics in a nonroutine context from student life	individual written response after a whole-class introduction
2. **Supermarket Carts**	45-minute problem illustrating an application of the mathematics; it is a nonroutine context from adult life; an approach to the task is to be formulated	individual written response after a whole-class introduction
3. **Assessing Logo Design**	45-minute task involving an illustrative application of mathematics in a nonroutine context from adult life	individual written response
4. **Designing a Staircase**	45-minute design task involving applied power over a nonroutine context from adult life; open-ended	individual written response
5. **Packaging a Soda Bottle**	60-minute design task, applied power; nonroutine mathematical connections; adult-life context	individual written response after a discussion in pairs
6. **Wheelchair Access**	45-minute design task, applied power in a nonroutine context from student life; open-ended	individual written response
7. **Kidney Stones**	45-minute exercise; adult-life context; applied power	individual written response after a discussion in pairs

* For explanations of terms that may be unfamiliar, see the Glossary, and the *Dimensions of Balance* table in the Introduction.

High School Package 1

Mathematical Content	Mathematical Processes
Number and Quantity: involving mainly estimation and associated computation	formulation of the problem and interpretation of the results roughly match the manipulation demands of the computation
Number and Quantity: structural analysis, leads into algebra, formulating functional relationships in symbolic form	formulation of the approach selection of a numerical or structural approach, manipulation and transformation of numerical and algebraic forms
Functions in a geometric context: involving both specific and general formulation of linear and quadratic relationships arising from scaling transformations	interpretation and evaluation of the given student responses leads to formulation of appropriate reasoning; some associated manipulation
Geometry: involves mainly estimation and computation of number and quantity in finding solutions that satisfy three constraints, one of which is the slope	balance of formulation of the approach, manipulation, interpretation of the trial results and their evaluation; strategy involves conjecture and check
Geometry, Space, and Shape, with Number: visualization of the form of the net for cuboid and hexagonal prism boxes; measurement of given figure; computation with Pythagorean theorem	formulation of the approach; manipulation through measurement, calculation, and sketching
Geometry, Space, and Shape, with Number: visualization of forms for the ramp; computation of slope constraints	formulation of possible designs; transformation of constraints into dimensions; communication about the design and how it meets the conditions
Data, Statistics, and Probability: combining probabilities by addition and multiplication, with circle chart	manipulation and interpretation of the data given, formulation of the standard method for combining probabilities

Expanded Table of Contents

Short Tasks	Task Type	Circumstances of Perfor
8. Lightning	15-minute problem involving applied power over a nonroutine context related to student life; tightly structured	individual written response
9. Homework, TV, and Sleep	15-minute exercise in a context from student life	individual written response
10. Wooden Water Tanks	15-minute problem involving applied power in a context from adult life	individual written response
11. Where's the Misprint?	15-minute problem involving applied power in a nonroutine context from adult life	individual written response
12. The Knockout	15-minute exercise; an illustrative application of probability	individual written response
13. Shadows	15-minute problem involving an illustrative application in a nonroutine context from student life	individual written response
14. Something's Fishy	15-minute task to prepare advice; applied power from nonroutine math in a context from adult life	individual written response
15. Miles of Words	15-minute problem involving an illustrative application in a nonroutine context from adult life	individual written response

High School Package 1

Mathematical Content	Mathematical Processes
Number and Quantity: a geometric problem involving location from a nonroutine coordinate representation	transformation and manipulation is the main load, with some interpretation of information given or calculated; almost an exercise
Handling scatter plot data	interpretation and formulation of scatter plot data, with associated manipulation
Number and Quantity applied to the volume of a cylinder of given dimensions, embraces measurement	formulation and consequent transformation of the expressions
Number: recognizing and correcting mismatched proportions of votes	evaluation of information provided and formulation of possible corrections, with associated manipulation
Combining probabilities	manipulation of probability computations
Function in a geometric context: involving graphical and algebraic representation of relationships in similar triangles	formulation and transformation of the relationships involved
Data, Statistics, and Probability: capture-recapture models for estimating populations	formulation and manipulation of the models, together with communication of results in a report
Creation, estimation, and application of a rate	interpretation, formulation, and manipulation are evenly balanced

Task 4

Overview

Use the concept of slope in an applied context involving staircases.

Operate with an inequality that limits the overall size of an allowable stair step.

Figure out a choice of rise and run that meets given requirements on stair size and slope.

Designing a Staircase

Long Task

Task Description

The task presents guidelines on step size and steepness allowable in staircases. The students are then asked to design a staircase that joins one floor with another floor 11 feet above it. The main job is to determine how many steps are required, and what size they must be.

Assumed Mathematical Background

Students should have done some work with the concept of slope of a line thought of as the (vertical) rise divided by the (horizontal) run of any section of the line.

Core Elements of Performance

- use the concept of slope (rise over run) in the setting of a staircase of repeated equal steps
- work with inequalities that specify the minimum and maximum allowable slope
- work with inequalities that specify minimum and maximum step size (where step size is measured as twice the rise plus the run)
- find the dimensions of a step that is within the guidelines and that can be used to span a given vertical distance

Circumstances

Grouping:	Students complete an individual written response.
Materials:	ruler and calculator
Estimated time:	45 minutes

Name Date

Designing a Staircase

This problem gives you the chance to

- ***design a staircase that meets certain guidelines***
- ***use the concept of slope in a practical situation***

Design a staircase that has a total rise of 11 feet and that meets the design guidelines given below.

Communicate your design decisions clearly: how many risers and treads are there, and what size are they?

Include your calculations.

Show how each design guideline is met.

Design Guidelines

- The slope of the staircase must be between 0.55 and 0.85.
- Twice the rise plus the run must be between 24 and 25 inches.
- There can be no irregular steps: Each step must be the same size.

Some useful terms

tread: the horizontal part of a step

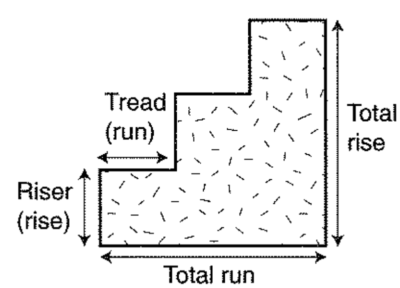
run: the length of the tread

riser: the vertical part of a step

rise: the height of the riser

slope: a measure of the steepness of a staircase found by dividing the riser height (rise) by the tread length (run):

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$



A Sample Solution

Task 4

There are many approaches to this problem. One is to guess a number of steps, say 20 steps. Since the stairs must rise 11 feet = 132 inches, each riser is $132 \div 20 = 6.6$ inches in height. It is required that twice the riser (13.2 inches) plus the tread must be between 24 and 25 inches. Let's choose 24.5 inches. This makes the tread 11.3 inches. What slope is this? It is $6.6 \div 11.3 \approx 0.58$. This is within the required limits $0.55 \leq \text{slope} \leq 0.85$. So this design works.

Summary: There are 20 steps, each with the dimensions riser = 6.6 inches and tread = 11.3 inches. The slope is about 0.58.

Another Sample Solution

Here is a more systematic solution with more of the reasoning supplied:

Use the notation m = slope, R = riser height (in inches), and T = tread length (in inches).

Since slope $= m = \frac{\text{rise}}{\text{run}} = \frac{\text{riser height}}{\text{tread length}} = \frac{R}{T}$, the relation $R = mT$ always holds.

Choose an arbitrary slope m for the staircase within the given range, say $m = 0.75$. Then $R = 0.75T$.

The requirement "twice the rise plus the run must be between 24 and 25 inches" can be expressed as: 24 inches $\leq 2R + T \leq 25$ inches. Substituting for R and solving for T, we get:

$24 \leq 2(0.75T) + T \leq 25$
$24 \leq 1.5T + T \leq 25$
$24 \leq 2.5T \leq 25$
$9.6 \leq T \leq 10$

Since $R = 0.75T$, 7.2 inches $\leq R \leq$ 7.5 inches.

The steps altogether must rise 11 feet (or 132 inches), and the individual steps must all be the same size. This means $132 \div R$ = a whole number. Checking the values of R at the two extremes:

132 inches ÷ 7.2 inches = 18.3 steps and 132 inches ÷ 7.5 inches = 17.6 steps.

Task 4

That means our staircase must have 18 steps. Since all the steps must be the same height, $R = 132 \div 18 = 7\frac{1}{3}$ inches, and $T = \frac{R}{0.75} = 9\frac{7}{9}$ inches.

Summary: There are 18 steps, each with the dimensions $R = 7\frac{1}{3}$ inches, and $T = 9\frac{7}{9}$ inches. The slope is 0.75.

We can double-check that our staircase meets the guideline 24 inches $\leq 2R + T \leq$ 25 inches: $2\left(7\frac{1}{3}\right) + 9\frac{7}{9} = 24\frac{4}{9}$ inches.

Since 24 inches $\leq 24\frac{4}{9}$ inches $\leq$ 25 inches, the staircase is within the guidelines.

Still Another Sample Solution

A more thorough treatment comes through starting with a number N of steps, finding the required riser height $R = 132 \div N$, finding the maximum and minimum tread length T for this value of R using the requirement 24 inches $\leq 2R + T \leq$ 25 inches, and computing the slope for each of these two tread lengths. This can be done for all the values of N that lead to acceptable slopes, and the result can be put in a table.

The key is in the computed slope. Those slopes that fall within the limits of 0.55 and 0.85 are entered in bold. These represent possible staircases. (Values are rounded off.)

N	$R = \frac{132}{N}$	$T = 24 - 2R$	slope $= \frac{R}{T}$	$T = 25 - 2R$	slope $= \frac{R}{T}$
16	8.25	7.5	1.1	8.5	0.97
17	7.76	8.48	0.92	9.48	**0.82**
18	7.33	9.34	**0.78**	10.34	**0.71**
19	6.95	10.1	**0.69**	11.1	**0.63**
20	6.6	10.8	**0.61**	11.8	**0.56**
21	6.29	11.42	**0.55**	12.42	0.51

The two extremes of tread length can be specified further:

The shortest tread comes for $N = 17$ steps, $R = 7.76$ inches. We can just set $\frac{R}{T} = 0.85$, the maximum allowed slope, and get $T \approx 9.13$ inches. As a check, $2R + T = 15.52 + 9.13 \approx 24.65$, which is less than the maximum of 25.

The longest tread comes for $N = 21$ steps, $R = 6.29$ inches. Here we can set $\frac{R}{T} = 0.55$, the minimum allowed slope, and get $T \approx 11.4$ inches.

Using this Task

Task 4

If you are using this task as part of a formal system of assessment, it should be presented to students with standardized instructions. For such purposes give each student a copy of the task and remind them that today is an assessment of their individual work.

For informal classroom use, students may work in pairs or in groups. It would be useful to collate student responses in a table such as that shown on the previous page. Here students would have an excellent example of a task that has more than one correct answer.

Extensions

Designing a Staircase can be extended by asking students to generate a number of acceptable solutions. Students may want to explore the use of slope in the context of wheelchair access. The Balanced Assessment task *Wheelchair Access* (Task 6), invites students to create access in accordance with the regulations laid out in the *Americans with Disabilities Act.*

Task 4

Characterizing Performance

This section offers a characterization of student responses and provides indications of the ways in which the students were successful or unsuccessful in engaging with and completing the task. The descriptions are keyed to the *Core Elements of Performance*. Our global descriptions of student work range from "The student needs significant instruction" to "The student's work meets the essential demands of the task." Samples of student work that exemplify these descriptions of performance are included below, accompanied by commentary on central aspects of each student's response. These sample responses are *representative;* they may not mirror the global description of performance in all respects, being weaker in some and stronger in others.

The characterization of student responses for this task is based on these *Core Elements of Performance:*

1. Use the concept of slope (rise over run) in the setting of a staircase of repeated equal steps.
2. Work with inequalities that specify the minimum and maximum allowable slope.
3. Work with inequalities that specify minimum and maximum step size (where step size is measured as twice the rise plus the run).
4. Find the dimensions of a step that is within the guidelines and that can be used to span a given vertical distance.

Descriptions of Student Work

The student needs significant instruction.

Typically the student works in a way that is unsystematic and seemingly arbitrary. It is as if the student knows that the task entails some sort of manipulation, but is unclear as to the theory that might drive these manipulations.

Student A

This student shows that he knows how to convert feet to inches. This is a useful start, but it is not clear why the student starts with 17.6 steps.

Task 4

The student needs some instruction.

Typically the student attempts to satisfy some but not all of the guidelines. The student focuses on one constraint, but the attempt is not sufficiently sustained to arrive at a specified model.

Student B

This student shows that she recognizes the importance of slope in this task. The student's approach to the problem is dominated by a search for a $\frac{\text{rise}}{\text{run}}$ that works. Notice the unequal equals that the student builds into her response when she writes something such as $\frac{55}{100} = \frac{85}{100}$. Setting the lower and upper values of the slope equal to each other is not helpful. It is almost as if the student hopes that setting up an "equation" might lead to possible values that will satisfy the constraints of the task. The procedure simply leads the student back to lower and upper values of slope. The student's response lies within the slope requirements but not within the second constraint, which has been ignored.

The student's work needs to be revised.

Aspects of the task are complete. The student typically proceeds by trial and error. This approach is quite time-consuming, but the student's attempt is sustained and delivers a possible solution. Typically the student does not show that each guideline has been met.

Student C

This student shows that she can use slope in a practical situation, but does not succeed in meeting each guideline. The student shows a consolidated understanding of slope. In addition, she shows that she makes a sustained attempt to meet each of the guidelines.

The student's work meets the essential demands of the task.

All core elements of performance are present. The student presents a design that is within each regulation. The student uses a diagram and explanations to show that each guideline is met.

Student D

This student shows that he can apply slope in a practical situation. The student figures out an accurate solution, and shows how each of the guidelines is met.

Student A

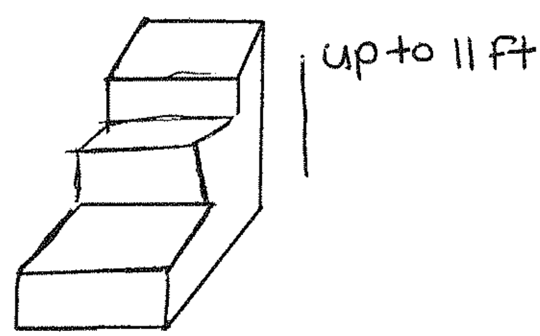
Staircase

11 feet X 12 inches = 132 inches

$\frac{132 \text{ inches}}{17.6 \text{ steps}}$ = 7.5 inches each - tread

The hight of the riser of each staircase will have to be 5.5 inches each and the

126 steps



Student B

$$\frac{55}{100} = \frac{85}{100}$$

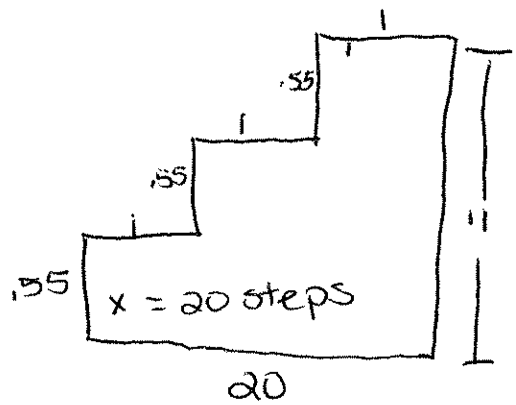
$$\frac{11}{20} = \frac{17}{20}$$

$$\frac{5.5}{10} = \frac{8.5}{10}$$

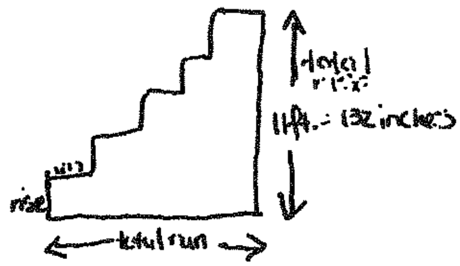
$$\frac{2.75}{5} = \frac{4.25}{5}$$

$$\frac{1.375}{2.5} = \frac{2.125}{2.5}$$

$$\frac{.55}{1} = \frac{0.85}{1}$$



Student C



total rise = 132 inch.

rise/run = between .55 + .85

2 X rise + run = between 24 + 25 inch.

1) total rise / total run = .55   132/y = .55   y = 240 inches = total run

132 inch / 240 inch = 15/27.2   15X2 + 27.2 = 57.2 (too high)

rise/run   132/240 = 4/7.27   4X2 + 7.27 = 15.3 (too low)   132/240 = 8/14.5   8X2 + 14.5 = 30.5 (too high)

132/240 = 6/10.9   6X2 + 10.9 = 22.9 too low   132/240 = 7/12.7   7X2 + 12.7 = 26.7 too high

rise/run   132 inch / 188.57 inch = .7 (slope) yes between .55 - .85   132 inch / 188.57 inch = 7 inch / 10 inch   7(2) + 10 = 24 inch. yes (between 24-25 inch)

riser height (rise) = 7 inch

tread length (run) = 10 inch

132 inch / 7 inch = almost 19 risers needed

132/19 = 6.95 inch = rise   188.57 inch / 10 inch = almost 19 treads needed

riser height (rise) = 6.95 inch.

tread length (run) = 9.92 inch.   188.57/19 = 9.92 inch = tread

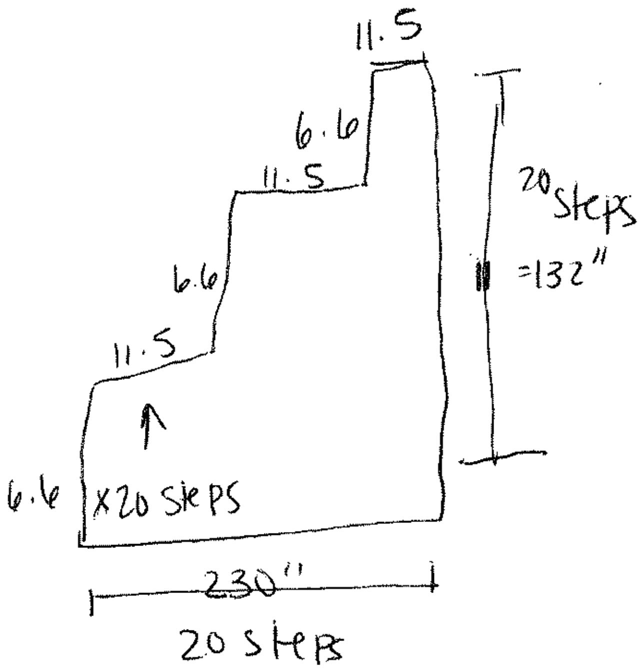
6.95 inch rise / 9.92 inch run = .7 = slope yes between .55 + .85

6.95 X2 + 9.92 = 23.8

NO (Not between 24-25)

WRONG

Student D



slopes

0 .55 = 55/100

11/20

5.5/10

2.5/5

.55/1

0.85 = 85/100

17/20

8.5/10

4.25/5

.85/1

Guidelines

Slope = 6.6/11.5   Slope = 0.55 > 0.85

Slope = 0.573913

6.6 × 2 = 13.2

13.2 + 11.5 = 24.7   twice "rise" plus "run" 24 > 25