Colorado
Academic Standards

Mathematics

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

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Prepared Graduates:

MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.NBT.A. Number & Operations in Base Ten: Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of algorithms may be used.

Evidence Outcomes

Students Can:

1. Use place value understanding to round whole numbers to the nearest 10 or 100. (CCSS: 3.NBT.A.1)
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 3.NBT.A.2)
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations. (CCSS: 3.NBT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly exhibit understanding of a variety of strategies when performing multi-digit arithmetic. (Personal Skills: Adaptability/Flexibility)
2. Demonstrate place value understanding by precisely referring to digits according to their place value. (MP6)
3. Recognize and use place value and properties of operations to structure algorithms and other representations of multi-digit arithmetic. (MP7)

Inquiry Questions:

1. How is rounding whole numbers to the nearest 10 or 100 useful?
2. Do different strategies for solving lead to different answers when we add or subtract? Why or why not?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 2, students use place value understanding and properties of operations to add and subtract fluently within 100.
3. This expectation connects to other ideas in Grade 3: (a) an understanding of multiplication, (b) knowing the relationship between multiplication and division, and (c) the concept of area and its relationship to multiplication and division.
4. In Grade 4, students generalize place value understanding for multi-digit whole numbers and use that understanding and the properties of operations to perform multi-digit arithmetic.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.NF.A. Number & Operations—Fractions: Develop understanding of fractions as numbers.

Evidence Outcomes

Students Can:

1. Describe a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$. (CCSS: 3.NF.A.1)
2. Describe a fraction as a number on the number line; represent fractions on a number line diagram. (CCSS: 3.NF.A.2)
   a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. (CCSS: 3.NF.A.2.a)
   b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. (CCSS: 3.NF.A.2.b)
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (CCSS: 3.NF.A.3)
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (CCSS: 3.NF.A.3.a)
   b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. (CCSS: 3.NF.A.3.b)
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express* 3 *in the form* $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and* 1 *at the same point of a number line diagram.* (CCSS: 3.NF.A.3.c)
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (CCSS: 3.NF.A.3.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly describe fractions both as parts of other numbers but also as numbers themselves. (Personal Skills: Adaptability/Flexibility)
2. Analyze and use information presented visually (for example, number lines, fraction models, and diagrams representing parts and wholes) that support an understanding of fractions as numbers. (Entrepreneurial Skills: Literacy/Reading)
3. Reason about the number line in a new way by understanding and using fractional parts between whole numbers. (MP2)
4. Critique the reasoning of others when comparing fractions that may refer to different wholes. (MP3)
5. Use the structure of fractions to locate and compare fractions on a number line. (MP7)

Inquiry Questions:

1. How does the denominator of a unit fraction connect to the number of unit fractions that must be added to make a whole?
2. When the numerators of two different fractions are the same, how can the denominators be used to compare them?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students (a) relate addition and subtraction to length, (b) measure and estimate lengths in standard units, and (c) reason with shapes and their attributes, including partitioning circles and rectangles into halves, thirds, and fourths.
3. In Grade 3, this expectation connects to the solving of problems involving measurement and estimation of intervals of time, liquid volumes, and mass of objects and is further supported by the expectation to represent and interpret data.
4. In Grade 4, students build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers and extend their understanding of fraction equivalence and ordering. In Grade 6, students apply and extend previous understandings of numbers (including fractions) to the system of rational numbers.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

3.OA.A. Operations & Algebraic Thinking: Represent and solve problems involving multiplication and division.

Evidence Outcomes

Students Can:

1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7. (CCSS: 3.OA.A.1)
2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$. (CCSS: 3.OA.A.2)
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (see Appendix, Table 2) (CCSS: 3.OA.A.3)
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \_ \div 3$, $6 \times 6 =?$ (CCSS: 3.OA.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving multiples and parts using multiplication and division. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of missing numbers in equations by using the relationship between multiplication and division. (MP1)
3. Reason abstractly about numbers of groups and the size of groups to make meaning of the quantities involved in multiplication and division. (MP2)
4. Use arrays to represent whole-number multiplication and division problems. (MP4)

Inquiry Questions:

1. How can an array be decomposed in a way that connects it to known multiplication facts? How can arrays be used to write and solve multiplication problems?
2. How can the area and one side of a rectangle be used to write and solve a division problem?
3. How could the number of dots in an array be counted without counting them one by one?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. In Grade 3, this expectation connects to understanding properties of multiplication, the relationship between multiplication and division, and to fluently multiplying and dividing within 100.
4. In Grade 4, students (a) use the four operations with whole numbers to solve problems, (b) build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers, and (c) solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. In Grade 5, students apply and extend previous understandings of multiplication and division to multiply and divide fractions.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.OA.B. Operations & Algebraic Thinking: Apply properties of multiplication and the relationship between multiplication and division.

Evidence Outcomes

Students Can:

5. Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If* $6 \times 4 = 24$ *is known, then* $4 \times 6 = 24$ *is also known. (Commutative property of multiplication.)* $3 \times 5 \times 2$ *can be found by* $3 \times 5 = 15$*, then* $15 \times 2 = 30$*, or by* $5 \times 2 = 10$*, then* $3 \times 10 = 30$*. (Associative property of multiplication.) Knowing that* $8 \times 5 = 40$ *and* $8 \times 2 = 16$*, one can find* 8×7 *as* $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$*. (Distributive property.)* (CCSS: 3.OA.B.5)
6. Interpret division as an unknown-factor problem. *For example, find* $32 \div 8$ *by finding the number that makes* 32 *when multiplied by* 8*.* (CCSS: 3.OA.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly work with different but related arrangements of factors and products or dividends, divisors, and quotients. (Personal Skills: Adaptability/Flexibility)
2. Use properties of operations to argue for or against the equivalence of different expressions. (MP3)
3. Be specific with explanations and symbols when describing operations using multiplication and division. (MP6)
4. Use the relationship between multiplication and division to rewrite division problems as multiplication. (MP7)

Inquiry Questions:

1. What are all of the equations that can be written to represent the relationship between the area of a (specific) rectangle and its side lengths?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. This expectation connects to other ideas in Grade 3: (a) multiplication and division within 100, (b) solving problems involving the four operations and identifying and explaining patterns in arithmetic, (c) understanding properties of multiplication and the relationship between multiplication and division, and (d) understanding concepts of area and the relationship to multiplication and division.
4. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic.

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Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

3.OA.C. Operations & Algebraic Thinking: Multiply and divide within 100.

Evidence Outcomes

Students Can:

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. (CCSS: 3.OA.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Efficiently solve multiplication and division problems by using facts committed to memory. (Professional Skills: Task/Time Management)
2. Recognize the relationship between skip counting and the solutions to problems involving multiplication and division. (MP7)

Inquiry Questions:

1. How can I use multiplication facts that I know to solve multiplication problems I do not yet know? (for example, using $5 \times 4 + 2 \times 4$ to solve 7×4)?
2. How can I use models and strategies to show what I know about multiplication?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. In Grade 3, this expectation connects with representing and solving problems involving the four operations.
4. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic, solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and gain familiarity with factors and multiples.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP6. Attend to precision.

Grade Level Expectation:

3.OA.D. Operations & Algebraic Thinking: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Evidence Outcomes

Students Can:

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This evidence outcome is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order of operations when there are no parentheses to specify a particular order.) (CCSS: 3.OA.D.8)
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that* 4 *times a number is always even, and explain why* 4 *times a number can be decomposed into two equal addends.* (CCSS: 3.OA.D.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving the four operations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Explain patterns in arithmetic. (MP3)
3. Mathematically model changes in quantities described in real-world contexts using the appropriate numbers, operations, symbols, and letters to represent unknowns. (MP4)
4. Complement arithmetic strategies with mental computation and estimation to assess answers for accuracy. (MP6)

Inquiry Questions:

1. How can a visual model support making sense of and solving word problems?
2. How can the patterns in addition and/or multiplication tables help predict probable solutions to a given problem?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students represent and solve one- and two-step word problems involving addition and subtraction.
3. This expectation connects to several ideas in Grade 3: (a) representing and solving problems involving multiplication and division, (b) multiplying and dividing within 100, (c) solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects, and (d) using concepts of area and relating area to multiplication and to addition.
4. In Grade 4, students use the four operations with whole numbers to solve problems.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

3.MD.A. Measurement & Data: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Evidence Outcomes

Students Can:

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. (CCSS: 3.MD.A.1)
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (This excludes compound units such as cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (This excludes multiplicative comparison problems, such as problems involving notions of "times as much." See Appendix, Table 2.) (CCSS: 3.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use units of measurement appropriate to the type and magnitude of the quantity being measured. (Professional Skills: Information Literacy)
2. Make sense of problems involving measurement by building on real-world knowledge of time and objects and an understanding of the relative sizes of units. (MP1)
3. Represent problems of time and measurement with equations, drawings, or diagrams. (MP4)
4. Use appropriate measures and measurement instruments for the quantities given in a problem. (MP5)

Inquiry Questions:

1. How can elapsed time be modeled on a number line to support the connection to addition and subtraction?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units and work with time and money.
3. In Grade 3, this expectation connects to developing an understanding of fractions as numbers, solving problems involving the four operations, and identifying and explaining patterns in arithmetic.
4. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

3.MD.B. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent* 5 *pets.* (CCSS: 3.MD.B.3)
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (CCSS: 3.MD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze data to distinguish the factual evidence offered, to reason about judgments, to draw conclusions, and to speculate about ideas the data represents. (Entrepreneurial Skills: Literacy/Reading)
2. Abstract real-world quantities into scaled graphs. (MP2)
3. Model real-world quantities with statistical representations such as bar graphs and line graphs. (MP4)

Inquiry Questions:

1. How can working with pictures and bar graphs connect mathematics to the world around us?
2. How does changing the scale of a bar graph or line plot change the appearance of the data?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 2, students represent and interpret length by measuring objects, make line plots, and use picture and bar graphs to represent categorical data.
3. In Grade 3, this expectation supports developing an understanding of fractions as numbers.
4. In Grade 4, students represent and interpret data by making line plots representing fractional measurements and solving addition and subtraction problems using information presented in line plots.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.MD.C. Measurement & Data: Geometric measurement: Use concepts of area and relate area to multiplication and to addition.

Evidence Outcomes

Students Can:

5. Recognize area as an attribute of plane figures and understand concepts of area measurement. (CCSS: 3.MD.C.5)
   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.C.5.a)
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. (CCSS: 3.MD.C.5.b)
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). (CCSS: 3.MD.C.6)
7. Use concepts of area and relate area to the operations of multiplication and addition. (CCSS: 3.MD.C.7)
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 3.MD.C.7.a)
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (CCSS: 3.MD.C.7.b)
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (CCSS: 3.MD.C.7.c)
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. (CCSS: 3.MD.C.7.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Defend calculations of area using multiplication and by tiling the area with square units and comparing the results. (MP3)
2. Understand how to use a one-dimensional measurement tool, like a ruler, to make two-dimensional measurements of area. (MP5)
3. Be precise by describing area in square rather than linear units. (MP6)
4. Use areas of rectangles to exhibit the structure of the distributive property. (MP7)

Inquiry Questions:

1. Given three pictures of different rectangles with unknown dimensions, how can you determine which rectangle covers the most area?
2. How does computing the area of a rectangle relate to closed arrays?
3. How can the area of an E-shaped or H-shaped figure be calculated?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units and reason with shapes and their attributes.
3. This expectation connects to other ideas in Grade 3: (a) recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures, (b) applying properties of multiplication and the relationship between multiplication and division, and (c) solving problems involving the four operations and identifying and explaining patterns in arithmetic.
4. In Grade 4, students solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit. In Grade 5, students relate volume to multiplication and to addition and also extend previous understandings of multiplication and division to multiply and divide fractions.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.

Grade Level Expectation:

3.MD.D. Measurement & Data: Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Evidence Outcomes

Students Can:

8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS: 3.MD.D.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of the relationship between area and perimeter by calculating both for rectangles of varying sizes and dimensions. (MP1)
2. Model perimeters of objects in the world with polygons and the sum of their side lengths. (MP4)

Inquiry Questions:

1. What are all the pairs of side lengths that can create a rectangle with the same area, such as 12 square units?
2. Is it possible for two rectangles to have the same area but different perimeters?
3. Is it possible for two rectangles to have the same perimeter but different areas?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units.
3. In Grade 3, this expectation connects to understanding concepts of area, relating area to multiplication and to addition, and solving problems involving the four operations.
4. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.G.A. Geometry: Reason with shapes and their attributes.

Evidence Outcomes

Students Can:

1. Explain that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (CCSS: 3.G.A.1)
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into* 4 *parts with equal area, and describe the area of each part as* $\frac{1}{4}$ *of the area of the shape.* (CCSS: 3.G.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Work with others to name and categorize shapes. (Civic/Interpersonal Skills: Collaboration/Teamwork)
2. Analyze, compare, and use the properties of geometric shapes to classify them into abstracted categories and describe the similarities and differences between those categories. (MP2)
3. Convince others or critique their reasoning when deciding if a shape belongs to certain categories of polygons. (MP3)
4. Decompose geometric shapes into polygons of equal area. (MP7)

Inquiry Questions:

1. Can you draw a quadrilateral that is not a rhombus, rectangle, or square?
2. (Given two identical squares) Divide each of these squares into four equal parts, but in different ways. If you compare a part of one with a part of the other, are their areas the same? How do you know?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 2, students reason with shapes and their attributes.
3. In Grade 3, this expectation connects to developing an understanding of fractions as numbers.
4. In Grade 4, students draw and identify lines and angles and also classify shapes by properties of their lines and angles.