Colorado
Academic Standards

Mathematics

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.

Grade Level Expectation:

2.NBT.A. Number & Operations in Base Ten: Understand place value.

Evidence Outcomes

Students Can:

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: (CCSS: 2.NBT.A.1)
   a. 100 can be thought of as a bundle of ten tens — called a "hundred." (CCSS: 2.NBT.A.1.a)
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (CCSS: 2.NBT.A.1.b)
2. Count within 1000; skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.A.2)
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS: 2.NBT.A.3)
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (CCSS: 2.NBT.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Abstract 10 ones into a single conceptual object called a ten and abstract 100 ones or 10 tens into a single conceptual object called a hundred. (MP2)
2. Compose, decompose, and compare three-digit numbers according to their base-ten structure. (MP7)

Inquiry Questions:

1. How many hundreds are in the number "four hundred five"? How do you know? How many tens are in the number "four hundred five? How do you know?
2. How many times do you need to skip count by 5s to count as far as skip counting by 10s once?
3. How many times do you need to skip count by 10s to count as far as skip counting by 100 once?
4. Why is any two-digit number that starts with 5 always larger than a two-digit number that starts with 3?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students understand place value for two-digit numbers.
3. In Grade 2, this expectation connects with using place value understanding and properties of operations to add and subtract and with working with equal groups of objects to gain foundations for multiplication.
4. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP7. Look for and make use of structure.

Grade Level Expectation:

2.NBT.B. Number & Operations in Base Ten: Use place value understanding and properties of operations to add and subtract.

Evidence Outcomes

Students Can:

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.B.5)
6. Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.B.6)
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS: 2.NBT.B.7)
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS: 2.NBT.B.8)
9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.) (CCSS: 2.NBT.B.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Relate concrete or mental strategies for adding and subtracting within 100 to a written method. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of place value by modeling quantities with drawings or equations. (MP1)
3. Use the base-ten structure to add and subtract, composing and decomposing ones, tens, and hundreds as necessary. (MP7)

Inquiry Questions:

1. Why might it be helpful to view subtraction as an unknown addend problem? (e.g., $278+?=425$)
2. How might you rewrite $38+47+93+62$ to make it easier to solve? How do you know it is OK to rewrite it?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students use place value and properties of operations to make sense of the relationship between addition and subtraction.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction and fluently adding and subtracting within 20.
4. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic, including fluently adding and subtracting within 1000.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

2.OA.A. Operations & Algebraic Thinking: Represent and solve problems involving addition and subtraction.

Evidence Outcomes

Students Can:

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (see Appendix, Table 1) (CCSS: 2.OA.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Decontextualize word problems, use mathematics to solve, and then recontextualize to provide the answer in context. (MP2)
2. Represent situations in word problems using drawings and equations with symbols for unknown numbers. (MP4)

Inquiry Questions:

1. (Given a word problem) What is the unknown quantity in this problem?
2. (Given an addition or subtraction problem) How might you use a model to represent this problem?
3. Does the word "more" in a word problem always mean that you will use addition to solve the problem? Why or why not?

Coherence Connections:

1. This expectation represents the major work of the grade.
2. In Grade 1, students use place value understanding and properties of operations to represent and solve problems involving addition and subtraction.
3. This expectation connects with other ideas in Grade 2: (a) using place value understanding and properties of operations to add and subtract, (b) relating addition and subtraction to length, (c) working with time and money, and (d) representing and interpreting data.
4. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.

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Prepared Graduates:

MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

2.OA.B. Operations & Algebraic Thinking: Add and subtract within 20.

Evidence Outcomes

Students Can:

2. Fluently add and subtract within 20 using mental strategies. (See 1.OA.C.6 for a list of strategies.) By end of Grade 2, know from memory all sums of two one-digit numbers. (CCSS: 2.OA.B.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize those problems that can be solved mentally versus those that require the use of objects, diagrams, or equations. (MP5)
2. Add and subtract within 20 quickly, accurately, and flexibly. (MP6)

Inquiry Questions:

1. How can you use addition and subtraction facts you know to quickly determine facts that you don't know?
2. Why do you think it is important to know your addition and subtraction facts?

Coherence Connections:

1. This GLE represents major work of the grade.
2. In Grade 1, students use objects and drawings to add and subtract within 20 in preparation for fluency with mental strategies in Grade 2.
3. In Grade 2, this expectation connects with using place value understanding and properties of operations to add and subtract within 1000 and fluently add and subtract within 100.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.

Grade Level Expectation:

2.OA.C. Operations & Algebraic Thinking: Work with equal groups of objects to gain foundations for multiplication.

Evidence Outcomes

Students Can:

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. (CCSS: 2.OA.C.3)
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (CCSS: 2.OA.C.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explore the arrangement of objects and how some arrangements afford mathematical power to solve problems. (Entrepreneurial Skills: Creativity/Innovation)
2. Reason about what it means for numbers to be even and odd. (MP2)
3. Explain why a group of objects is even or odd and if a strategy for deciding works with any group of objects. (MP3)

Inquiry Questions:

1. What does it mean for a number to be even?
2. Do two equal addends always result in an even sum? Why or why not?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 1, students work with addition and subtraction equations.
3. In Grade 2, this expectation connects with understanding place value for three-digit numbers.
4. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

2.MD.A. Measurement & Data: Measure and estimate lengths in standard units.

Evidence Outcomes

Students Can:

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (CCSS: 2.MD.A.1)
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (CCSS: 2.MD.A.2)
3. Estimate lengths using units of inches, feet, centimeters, and meters. (CCSS: 2.MD.A.3)
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS: 2.MD.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Consider the correctness of another students' measurement in which they lined up three large and four small blocks and claimed a path was "seven blocks long." (MP3)
2. Choose between different measurement tools depending on the objects they need to measure. (MP5)
3. Determine when it is appropriate to estimate an object's length or when a more precise measurement is needed. (MP6)

Inquiry Questions:

1. What do the numbers on a ruler represent?
2. What is the more appropriate tool for measuring the length of your school hallway, a 1-foot ruler or a 25-foot measuring tape?
3. When is it appropriate to estimate length? When is it not appropriate?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students measure lengths indirectly and by iterating length units.
3. In Grade 2, this expectation connects with relating addition and subtraction to length and with representing and interpreting data.
4. In Grade 3, students (a) develop understanding of fractions as numbers, (b) solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects, and (c) use concepts of area and relate area to multiplication and to addition.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

2.MD.B. Measurement & Data: Relate addition and subtraction to length.

Evidence Outcomes

Students Can:

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.MD.B.5)
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1,2, ..., and represent whole-number sums and differences within 100 on a number line diagram. (CCSS: 2.MD.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize problems involving lengths and identify possible solutions. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Build on experiences with measurement tools to understand number lines as a more abstract tool for working with quantities. (MP2)
3. Use mathematical representations, like drawings and equations, to model scenarios described in word problems. (MP4)

Inquiry Questions:

1. When might it be necessary to measure parts of objects and then combine those parts together?
2. How is a number line like a ruler?

Coherence Connections:

1. This GLE represents major work of the grade.
2. In Grade 1, students add and subtract within 20 and express the length of an object as a whole number of length units.
3. In Grade 2, this expectation connects with measuring and estimating lengths in standard units and with representing and interpreting data.
4. In Grade 3, students develop an understanding of a fraction as a number on a number line.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP6. Attend to precision.

Grade Level Expectation:

2.MD.C. Measurement & Data: Work with time and money.

Evidence Outcomes

Students Can:

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS: 2.MD.C.7)
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have two dimes and three pennies, how many cents do you have?* (CCSS: 2.MD.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Tell and manage time to be both personally responsible and responsible to the needs of others. (Personal Skills: Personal Responsibility)
2. Make sense of word problems involving money. (MP1)
3. Recognize that time is a quantity that can be measured with different degrees of precision. (MP6)

Inquiry Questions:

1. If the time is 2:25, where would the minute hand be pointing on an analog clock?
2. Does the size of a coin indicate the value of the coin?
3. How is money like our base-ten number system, where it takes ten of one unit to make the next unit (ten ones makes a ten, ten tens make a hundred)? In what ways is it different?

Coherence Connections:

1. This GLE supports the major work of the grade.
2. In Grade 1, students tell and write time in hours and half-hours using analog and digital clocks.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction.
4. In Grade 3, students tell and write time to the nearest minute and measure time intervals in minutes.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

2.MD.D. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS: 2.MD.D.9)
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems (see Appendix, Table 1) using information presented in a bar graph. (CCSS: 2.MD.D.10)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Organize objects according to measures or categories to help make sense of problems. (MP1)
2. Organize measurement and categorical data into categories based on size or type so comparisons can be made between categories instead of between individual objects. (MP2)
3. Discuss ways in which bar graph orientation (horizontal or vertical), order, thickness, spacing, shading, colors, etc. make the graphs easier or more difficult to interpret. (MP5)

Inquiry Questions:

1. How is organizing objects by length measurements, rounded to the nearest unit, similar to and different from organizing objects by categories?
2. (Given a bar graph representation of up to four categories of animals) How many more birds are there than hippos? How many more giraffes would there need to be in order for the number of giraffes to equal the number of elephants?

Coherence Connections:

1. This GLE supports the major work of the grade.
2. In Grade 1, students organize, represent, and interpret data with up to three categories and compare how many more or less are in one category than another.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction and with relating addition and subtraction to length.
4. In Grade 3, students draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.

Grade Level Expectation:

2.G.A. Geometry: Reason with shapes and their attributes.

Evidence Outcomes

Students Can:

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (CCSS: 2.G.A.1)
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS: 2.G.A.2)
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. (CCSS: 2.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Demonstrate flexibility, imagination, and inventiveness in drawing shapes having specified attributes and in partitioning circles and rectangles into equal shares. (Entrepreneurial Skills: Risk Taking)
2. Explore various ways of partitioning shapes into equal shares, such as different methods for dividing a square into fourths, to understand that each partition, regardless of shape, represents an equal share of the square. (MP2)
3. Engage in spatial structuring by tiling rectangles with rows and columns of squares to build understanding of two-dimensional regions. (MP7)

Inquiry Questions:

1. How many different triangles can you draw where two of the sides have the same length?
2. (Given a rectangle) Can you divide this rectangle into three equal parts in more than one way?

Coherence Connections:

1. This GLE is in addition to the major work of Grade 2.
2. In Grade 1, students reason with shapes and their attributes, distinguish between defining and non-defining attributes, compose two-dimensional shapes, and partition circles and rectangles into halves and fourths.
3. In Grade 3, students develop understanding of fractions as numbers, use concepts of area and relate area to multiplication and to addition, and understand that shared attributes in different categories of shapes can define a larger category.