Colorado Academic Standards

Mathematics

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards (CCSSI, 2010). Across the curriculum at every grade, students are expected to consistently have opportunities to engage in each of the eight practices. The practices aligned with each Grade Level Expectation in the Colorado Academic Standards represent the *strongest potential* alignments between content and the practices, and are not meant to exclude students from engaging in the rest of the practices.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Math Practice MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Math Practice MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Math Practice MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Practice MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Math Practice MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Math Practice MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Math Practice MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1,2)$ with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

References

Common Core State Standards Initiative. (2010). *Standards for mathematical practice*. *http://www.corestandards.org/Math/Practice*

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. (For more on modeling, see Appendix: Modeling Cycle.)

How to Read the Colorado Academic Standards

CONTENT AREA
Grade Level, Standard Category

COLORADO Department of Education

Prepared Graduates:

The *PG Statements* represent concepts and skills that all students who complete the Colorado education system must master to ensure their success in postsecondary and workforce settings.

Grade Level Expectation:

The *GLEs* are an articulation of the concepts and skills for a grade, grade band, or range that students must master to ensure their progress toward becoming a prepared graduate.

Evidence Outcomes

The *EOs* describe the evidence that demonstrates that a student is meeting the GLE at a mastery level.

Academic Context and Connections

The *ACCs* provide context for interpreting, connecting, and applying the content and skills of the GLE. This includes the *Colorado Essential Skills*, which are the critical skills needed to prepare students to successfully enter the workforce or educational opportunities beyond high school embedded within statute (C.R.S. 22-7-1005) and identified by the Colorado Workforce Development Committee.

The ACCs contain information unique to each content area. Content-specific elements of the ACCs are described below.

Colorado Academic Standards
Content Area

Grade Level, Standard Category 2020 Colorado Academic Standards GLE Code

Academic Context and Connections in Mathematics:

Colorado Essential Skills and Mathematical Practices: These statements describe how the learning of the content and skills described by the GLE and EOs connects to and supports the development of the *Colorado Essential Skills* and *Standards for Mathematical Practice* named in the parentheses.

Inquiry Questions: The sample question that are intended to promote deeper thinking, reflection, and refined understandings precisely related to the GLE.

Coherence Connections: These statements relate how the GLE relates to content within and across grade levels. The first statement indicates if a GLE is *major*, *supporting*, or *additional* work of the grade. Between 65% and 85% of the work of each grade (with P-2 at the high end of that range) should be focused on the GLEs labeled as **major work**. The remainder of the time should focus on **supporting work** and **additional work**, where it can appropriately support and compliment students' engagement in major work. **Advanced outcomes**, marked with a (+), represent content best saved for upper-level math courses in a student's final three semesters of high school. The remaining statements describe how the GLE and EOs build from content learned in prior grades, connects to content in the same grade, and supports learning in later grades.

Prepared Graduates:

MP8. Look for and express regularity in repeated reasoning.

Preschool Learning and Development Expectation:

P.CC.A. Counting & Cardinality: Know number names and the count sequence.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

1. Count verbally or sign to at least 20 by ones.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Count and use numbers as they play with children.
2. Take advantage of every opportunity to count with children in a practical and authentic setting.

Examples of Learning/Children May:

1. Read stories, sing songs, and act out poems and finger plays that involve counting, numerals, and shapes.
2. Practice saying a sequence of number words.
3. Respond to the question, “What comes after four?” with “One, two, three, four … five!”

Coherence Connections:

1. This expectation represents major work of the grade.
2. Between 24–36 months, children say or sign some number words in sequence, starting with one, and understand that counting words are separate words, such as “one,” “two,” “three,” versus “onetwothree.”
3. In preschool, learning the counting sequence is part of learning progressions that go (a) from saying the counting words to counting out objects and (b) from speaking number words to writing base-ten numerals.
4. In kindergarten, students count to 100 by ones and tens and count forward from a given number.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

Preschool Learning and Development Expectation:

P.CC.B. Counting & Cardinality: Recognize the number of objects in a small set.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

2. Instantly recognize, without counting, small quantities of up to five objects and say or sign the number.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Hold five or fewer objects in a closed hand, then open it briefly for the child, close it again, and ask, "How many did you see?"
2. Quickly show children a card with five or fewer dots, then hide it and ask who can say how many dots they saw.
3. Ask children to place their hands where they can't see them, then show a small number on their fingers, then have the children check their work by looking at their hands.

Examples of Learning/Children May:

1. Play with a friend and say without counting, "I have five big rocks and you have five little rocks. We have the same."
2. Find fewer objects or objects in patterns (like two rows of 2 to make four) easier to subitize.

Coherence Connections:

1. This expectation supports the major work of the grade.
2. Between 36–60 months, children develop an understanding of what whole numbers mean and become increasingly able to quickly recognize the number of objects in a small set (known as subitizing).
3. In preschool, subitizing facilitates efficient counting.
4. In kindergarten, students count to determine the number of up to 20 arranged or up to 10 scattered objects.

Prepared Graduates:

MP6. Attend to precision.

Preschool Learning and Development Expectation:

P.CC.C. Counting & Cardinality: Understand the relationship between numbers and quantities.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

3. Say or sign the number names in order when counting, pairing one number word that corresponds with one object, up to at least 10.
4. Use the number name of the last object counted to answer “How many?” questions for up to approximately 10 objects.
5. Accurately count as many as five objects in a scattered configuration or out of a collection of more than five objects.
6. Understand that each successive number name refers to a quantity that is one larger.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Play age-appropriate games that involve counting spaces or objects.
2. Count to five from thumb to pinky on an open hand, then close the hand except for the pinky and ask, “How many fingers are still showing?” to see if a child answers one or five.
3. Help children count by pointing to objects or drawings of objects, then confirming the total by asking, “So how many are there altogether?”
4. Provide opportunities to count objects for lunch, such as plates, napkins, and cups.

Examples of Learning/Children May:

1. Match a group of 1 to 10 objects with written and spoken numbers.
2. Play simple games that match numbers to a movement of spaces on a game board.
3. Take a specified number of crackers from a bowl during snack time.

Coherence Connections:

1. This expectation represents major work of the grade.
2. Between 36–60 months, children coordinate verbal counting with objects by pointing at each object for each number word (known as one-to-one correspondence) and develop an understanding that the last number in the sequence represents how many in the group (known as cardinality).
3. In preschool, students connect the process of counting to a conceptual understanding of cardinality.
4. In kindergarten, students count to determine the number of objects using one-to-one correspondence and cardinality for up to 20 objects in a line or 10 scattered objects.

Prepared Graduates:

MP7. Look for and make use of structure.

Preschool Learning and Development Expectation:

P.CC.D. Counting & Cardinality: Compare numbers.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

7. Identify whether the number of objects in one group is more than, less than or the same as objects in another group for up to at least five objects.
8. Identify and use numbers related to order or position from first to fifth.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Have children group and order materials when cleaning up.
2. Describe quantities using vocabulary including more than, less than, and equal to.
3. Provide opportunities for children to count, group, and order objects and materials.
4. Put four counting chips inside a circle and one chip outside the circle, then ask, “Which has more, inside or outside? Which has fewer chips?”

Examples of Learning/Children May:

1. Count, group, and sort objects and materials.
2. Be able to express a preference for greater numbers of things (such as candy or toys) when comparing groups of different sizes.
3. Say phrases like, “There are more cookies in this box,” or “There are fewer pencils on that table than on this one.”
4. Identify which item is first, second, third, etc., when pointing to items or talking about events that are ordered.

Coherence Connections:

1. This expectation represents major work of the grade.
2. Between 36–60 months, children begin to count and compare same-size objects (with adult assistance) and begin to understand that the number of objects is independent of the size of the objects.
3. In kindergarten, students identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group for up to 10 objects. Students also compare two numbers between 1 and 10 presented as written numerals.

Prepared Graduates:

MP5. Use appropriate tools strategically.

Preschool Learning and Development Expectation:

P.CC.E. Counting & Cardinality: Associate a quantity with written numerals up to 5 and begin to write numbers.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

9. Associate a number of objects with a written numeral 0–5.
10. Recognize and, with support, write some numerals up to 10.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Play games with children where spinning a wheel with numbers or the number written on a card is associated with the need to count that number of objects or spaces.
2. Help a child write or trace using any writing tool the numeral corresponding to his or her age.
3. Support the use of a numeral by connecting it to a group of objects or a picture of objects to help students associate the numeral to a quantity.

Examples of Learning/Children May:

1. Match a group of 1 to 5 objects with written and spoken numbers.
2. Copy a printed numeral using their own handwriting.
3. Play games that involve matching numerals to numbers of objects, such as dots on cards.

Coherence Connections:

1. This expectation supports the major work of the grade.
2. Between 36–60 months, children develop an understanding that a written numeral represents a quantity and uses symbols, like tally marks, to represent numerals.
3. In preschool, work with numerals is still in its early stages. Writing numerals does not become a focus until kindergarten, but it can be done in preschool to support other work in mathematics and writing.
4. In kindergarten, students write numbers from 0 to 20 and associate a number of objects with the written numerals 0–20.

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Prepared Graduates:

MP4. Model with mathematics.

Preschool Learning and Development Expectation:

P.OA.A. Operations & Algebraic Thinking: Understand addition as adding to and understand subtraction as taking away from.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

1. Represent addition and subtraction in different ways, such as with fingers, objects, and drawings.
2. Solve addition and subtraction problems set in simple contexts. Add and subtract up to at least five to or from a given number to find a sum or difference up to 10.
3. With adult assistance, begin to use counting on (adding 1 or 2, for example) from the larger number for addition.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Use fingers on both hands to represent addition.
2. Ask a child with five crackers, “If you eat three of your crackers, how many will you have left?”
3. Ask “How many more?” questions, such as, “We have three children in this group. How many more children do we need to make a group of five?”

Examples of Learning/Children May:

1. Add a group of three and a group of two, counting “One, two three …” and then counting on “Four, five!” while keeping track using their fingers.
2. Take three away from five, counting “Five, four, three … two!” while keeping track using their fingers.
3. Say after receiving more crackers at snack time, “I had two and now I have four.”
4. Predict what will happen when one more object is taken away from a group of five or fewer objects, and then verify their prediction by taking the object away and counting the remaining objects.

Coherence Connections:

1. This expectation represents major work of the grade.
2. Between 36–60 months, children develop beginning understandings of adding and subtracting with the help of objects and adult support.
3. In preschool, students should work with small numbers and simpler problem subtypes (see Appendix, Table 1).
4. In kindergarten, students add and subtract within 10 using objects or drawings to represent problems and fluently add and subtract within 5.

Prepared Graduates:

MP8. Look for and express regularity in repeated reasoning.

Preschool Learning and Development Expectation:

P.OA.B. Operations & Algebraic Thinking: Understand simple patterns.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

4. Fill in missing elements of simple patterns.
5. Duplicate simple patterns in a different location than demonstrated, such as making the same alternating color pattern with blocks at a table that was demonstrated on the rug. Extend patterns, such as making an eight-block tower of the same pattern that was demonstrated with four blocks.
6. Identify the core unit of sequentially repeating patterns, such as color in a sequence of alternating red and blue blocks.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Provide everyday opportunities to explore numbers and patterns, such as setting the table with a cup, plate, and fork for each person.
2. Provide opportunities to observe naturally occurring patterns within the indoor and outdoor environments, such as looking at patterns in the bricks of a building or patterns in art and design.
3. Introduce songs and movement patterns where children can extend and grow the pattern.

Examples of Learning/Children May:

1. Use art materials and other objects to create or replicate patterns (e.g., weaving, stringing beads, stacking blocks, or drawing repeating pictures).
2. Recognize patterns in a story or song.
3. Identify two blocks, one red and one blue, as the core unit of a longer pattern using alternating red and blue blocks.
4. Sequence story cards to show beginning, middle, and end.

Coherence Connections:

1. This expectation supports the major work of the grade.
2. Between 36–60 months, children recognize and work with simple patterns (like ABAB) in different forms, such as patterns of objects, numbers, sounds, and movements.
3. In preschool, students may recognize and duplicate more complicated patterns, such as ABC, ABB, and AABB.
4. In kindergarten, pattern recognition is embedded in and focused on early numeracy, such as counting by tens, number composition/decomposition, making tens, describing attributes of objects, and classifying objects into categories.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

Preschool Learning and Development Expectation:

P.MD.A. Measurement & Data: Measure objects by their various attributes using standard and nonstandard measurement and use differences in attributes to make comparisons.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

1. Use comparative language, such as shortest, heavier, biggest, or later.
2. Compare or order up to five objects based on their measurable attributes, such as height or weight.
3. Measure using the same unit, such as putting together snap cubes to see how tall a book is.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Follow a pictorial recipe and let children measure, pour, and stir the ingredients while asking questions like, “How many cups of flour does the recipe show we need to put in the bowl?”
2. Provide opportunities for children to sort, classify and group household objects and materials.
3. Ask questions of measurement (e.g., “How many steps does it take to walk from the front door to your cubby?” or “How many blocks long is your arm?”).
4. Offer a variety of measuring tools and models, such as rulers, yardsticks, measuring tapes, measuring cups, scales, and thermometers. (Children may not use each of these correctly, but they are developing early understandings of how tools measure things.)
5. Provide opportunities for children to use non-standard measuring tools such as cubes, paperclips, blocks, etc.

Examples of Learning/Children May:

1. Sort objects by physical characteristics such as a color or size.
2. Group objects according to their size, using standard and nonstandard forms of measurement (e.g., height, weight, length, color, or brightness).
3. Explore various processes and units for measurement and begin to notice different results of one method or another.

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. Between 36–60 months, children develop an understanding that attributes can be described and compared in simple ways, such as one child being taller than another.
3. In preschool, this expectation connects with counting, comparing numbers, and comparing shapes.
4. In kindergarten, students describe multiple measurable attributes of an object and make direct comparisons of two objects with a measurable attribute in common.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

Preschool Learning and Development Expectation:

P.G.A. Geometry: Identify, describe, compare, and compose shapes.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

1. Name and describe shapes in terms of length of sides, number of sides, and number of angles/corners.
2. Correctly name basic shapes (circle, square, rectangle, triangle) regardless of size and orientation.
3. Analyze, compare, and sort two-and three-dimensional shapes and objects in different sizes. Describe their similarities, differences, and other attributes, such as size and shape.
4. Compose simple shapes to form larger shapes.

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Use a sensory table with various bowls, cups, or other containers to encourage activities with shapes and sorting.
2. Provide children with puzzles made of simple geometric shapes and encourage saying the names of shapes as they play.
3. Discuss geometric shapes in terms of their attributes, such as “This is a circle. It’s perfectly round with no bumps or corners. This is a triangle. It has three sides and three angles.”
4. Use a variety of lengths and angles in their shapes (such as scalene triangles, long and thin rectangles) as well as more common configurations of shapes (such as equilateral triangles).

Examples of Learning/Children May:

1. Match, sort, group, and name basic shapes found outside or in the classroom.
2. Use pattern tiles to make shapes out of other shapes, such as putting two squares side-by-side to make a non-square rectangle.
3. Put away blocks and/or tiles into different containers based on the number or length of sides.

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. Between 36–60 months, children start by recognizing circles and squares and then add triangles and other shapes. As understanding of shape develops, children identify sides and angles as distinct parts of shapes.
3. In preschool, this expectation connects with measuring and comparing objects by their attributes.
4. In kindergarten, students identify and describe squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.

Prepared Graduates:

MP6. Attend to precision.

Preschool Learning and Development Expectation:

P.G.B. Geometry: Explore the positions of objects in space.

Indicators of Progress

By the end of the preschool experience (approximately 60 months/5 years old), students may:

5. Understand and use language related to directionality, order, and the position of objects, including up/down and in front/behind.
6. Correctly follow directions involving their own position in space, such as "Stand up" and "Move forward."

Examples of High-Quality Teaching and Learning Experiences

Supportive Teaching Practices/Adults May:

1. Provide opportunities for conversation using everyday words to indicate space location, shape, and size of objects, saying things like, "You crawled under the picnic table, over the tree stump, and now you are in the tunnel slide!"
2. Help children organize toys, pointing out concepts such as "in," "on," and "beside."

Examples of Learning/Children May:

1. Use the vocabulary of geometry and position to describe shapes within the room and surrounding environment.
2. Understand relational directions, such as "Please put a mat under each plate."

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. Between 36–60 months, students develop spatial vocabulary and become able to follow directions involving their own position in space.
3. In preschool and early elementary, students work with shapes and their attributes in increasingly sophisticated ways over time.
4. In kindergarten, students describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, in front of, behind, and next to.

Prepared Graduates:

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

K.CC.A. Counting & Cardinality: Use number names and the count sequence.

Evidence Outcomes

Students Can:

1. Count to 100 by ones and by tens. (CCSS: K.CC.A.1)
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1). (CCSS: K.CC.A.2)
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects). (CCSS: K.CC.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that the number sequence from 1 to 9 repeats between the decade numbers, except in the spoken numbers between 10 and 20. (MP7)
2. Reason that counting to 100 by tens reaches the same number as can be counted repeatedly by ones. (MP8)

Inquiry Questions:

1. When might you want to count by tens instead of ones?
2. When might you want to start counting from a number other than one?
3. What number can we use to show we have nothing to count?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students understand that number words have a sequence and that the words are separate (not “onetwothree”).
3. In kindergarten, this expectation is key to several progressions of learning: (a) from saying the counting words to counting out objects, (b) from counting to counting on, and (c) from spoken number words to written base-ten numerals to base-ten system understanding.
4. In Grade 1, students extend the counting sequence to 120.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

K.CC.B. Counting & Cardinality: Count to determine the number of objects.

Evidence Outcomes

Students Can:

4. Apply the relationship between numbers and quantities and connect counting to cardinality. (CCSS: K.CC.B.4)
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (CCSS: K.CC.B.4.a)
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (CCSS: K.CC.B.4.b)
   c. Understand that each successive number name refers to a quantity that is one larger. (CCSS: K.CC.B.4.c)
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. (CCSS: K.CC.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Progress from thinking about numbers as the result of the process of counting to abstractly thinking about numbers as mental objects of their own—especially the quantity 10. (MP2)
2. Explain how the number reached when counting on is a relationship between the quantity started from and the quantity added. (MP3)
3. Make counting efficient by following rows, columns, or other patterns in a group of arranged objects. (MP7)

Inquiry Questions:

1. How is counting to five different from the number five?
2. What number is one larger than four? What number is one larger than seven?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students build conceptions of what whole numbers mean, of subitizing, of one-to-one correspondence between verbal counting and objects, and of cardinality.
3. In kindergarten, this expectation is key to several progressions of learning: (a) from saying the counting words to counting out objects, (b) from counting to counting on, and (c) from spoken number words to written base-ten numerals to base-ten understanding.
4. In Grade 1, students use their understanding of counting and cardinality to add and subtract within 20.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

Grade Level Expectation:

K.CC.C. Counting & Cardinality: Compare numbers.

Evidence Outcomes

Students Can:

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to 10 objects.) (CCSS: K.CC.C.6)
7. Compare two numbers between 1 and 10 presented as written numerals. (CCSS: K.CC.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make reasoned arguments about the relative sizes of groups, such as by matching objects of two groups and seeing which has extra objects, or by counting the objects in each group and seeing which has the number further in the counting sequence. (MP3)
2. Use precise language to describe why one quantity is less than, greater than, or equal to another, and avoid mixing and misusing different ways of quantifying such as dimension, weight, or magnitude. (MP6)

Inquiry Questions:

1. Other than counting, how might you decide whether one set has more objects than another?
2. Which is more, 3 small cookies or 2 big cookies? What makes this difficult to answer?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students build an understanding of same versus different numbers of items, numbers of objects versus their size, and ordering from first to fifth.
3. In kindergarten, this expectation is key to several progressions of learning: (a) from counting to counting on and (b) from comparison by matching to comparison by numbers to comparison involving adding and subtracting.
4. In Grade 1, students build an understanding of ten and place value with two-digit numbers. Students also organize data into categories and compare how many more or less are in one category than in another.

Prepared Graduates:

MP6. Attend to precision.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

K.NBT.A. Number & Operations in Base Ten: Work with numbers 11–19 to gain foundations for place value.

Evidence Outcomes

Students Can:

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: K.NBT.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Be precise in drawings, diagrams, and numerical recordings about objects or symbols that represent ones and objects or symbols that represent tens. (MP6)
2. See the structure of a number as composed of its base-ten units. (MP7)
3. Repeat the reasoning afforded by the uniformity of the base-ten system, where 10 copies compose 1 base-ten unit of the next highest value. (MP8)

Inquiry Questions:

1. Can you show the number 13 as ten ones and some more ones? How many more ones than tens are there?
2. In the number 11, what makes the "1" on the left different from the "1" on the right? Could you show this with objects or a diagram?
3. What would a number called "ten four" look like? What word do we usually say for this number?
4. Why might someone call the number 17 "ten seven?"

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students develop conceptions of addition and subtraction when adding to and taking away from small collections of objects.
3. In kindergarten, this expectation is part of a progression from comparison by spoken number words to written base-ten numerals to base-ten system understanding.
4. In Grade 1, students build an understanding of ten and place value with two-digit numbers.

Prepared Graduates:

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

Grade Level Expectation:

K.OA.A. Operations & Algebraic Thinking: Model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings.

Evidence Outcomes

Students Can:

1. Represent addition and subtraction with objects, fingers, mental images, drawings (drawings need not show details, but should show the mathematics in the problem), sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (CCSS: K.OA.A.1)
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. (CCSS: K.OA.A.2)
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$). (CCSS: K.OA.A.3)
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS: K.OA.A.4)
5. Fluently add and subtract within 5. (CCSS: K.OA.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of real-world situations involving addition and subtraction (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Mathematize a real-world situation, focusing on the quantities and their relationships rather than non-mathematical aspects of the situation. (MP4)
3. Act out adding and subtracting situations by representing quantities in the situation with objects, fingers, and math drawings. (MP5)
4. Use the equal sign consistently and appropriately. (MP6)

Inquiry Questions:

1. How could you show me adding 3 and 2?
2. How could you show me 3 take away 2?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In preschool, students represent addition and subtraction within 5 with fingers, objects, and drawings.
3. In kindergarten, this expectation is part of a progression involving addition and subtraction of increasingly large numbers and increasingly complex problem subtypes (see Appendix, Table 1).
4. In Grade 1, students understand properties of operations, the relationship between addition and subtraction, and add and subtract within 20.

Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

K.MD.A. Measurement & Data: Describe and compare measurable attributes.

Evidence Outcomes

Students Can:

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. (CCSS: K.MD.A.1)
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* (CCSS: K.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of their world by comparing and ordering objects by their attributes. (Entrepreneurial Skills: Inquiry/Analysis)
2. Be precise about meanings related to size when describing an object's height, weight, or other attribute. (MP6)

Inquiry Questions:

1. What does it mean for one object to be "bigger" than another?
2. If you are standing on a chair, how should your height be measured differently than if you were standing on the floor?
3. If an object is moved, does that change its size?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students develop conceptions of measurable attributes of objects and comparisons based on those attributes.
3. In kindergarten, this expectation can contribute to students' understandings of measurable attributes, comparison, and conservation of length, all of which connect to progressions in geometry, the number system, and to future work in ratio and proportion.
4. In Grade 1, students measure lengths directly and by iterating length units, and express the length of an object as a whole number of length units.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

K.MD.B. Measurement & Data: Classify objects and count the number of objects in each category.

Evidence Outcomes

Students Can:

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.) (CCSS: K.MD.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Group objects into categories to help make sense of problems. (MP1)
2. Abstract individual objects into new conceptual groups. (MP2)
3. Choose appropriate representations of objects and categories. (MP5)

Inquiry Questions:

1. How can numbers of objects be represented to make comparisons?
2. How can objects be categorized in different ways?
3. How can an object's attributes determine if it does not belong with other objects in a group?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In preschool, students use differences in attributes to make comparisons.
3. In kindergarten, this expectation supports the work of counting and comparing numbers and is part of a progression of learning how to analyze categorical data.
4. In Grade 1, students organize, represent, and interpret data with up to three categories.

Prepared Graduates:

MP4. Model with mathematics.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

K.G.A. Geometry: Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Evidence Outcomes

Students Can:

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. (CCSS: K.G.A.1)
2. Correctly name shapes regardless of their orientations or overall size. (CCSS: K.G.A.2)
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). (CCSS: K.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Describe the physical world from geometric perspectives, e.g., shape, orientation, and spatial relationships. (MP4)
2. Reflect an increasing understanding of shapes by using increasingly precise language to describe them. (MP6)
3. Sort shapes into categories (squares, circles, triangles, etc.) based on attributes of the shapes. (MP7)

Inquiry Questions:

1. For a given shape, what attributes make an example of that shape different from a non-example? For example, "Why is this shape (point to a square) a square, while this shape (point to a non-square) is not?"
2. What are the ways of describing where an object is?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students learn about circles, squares, triangles, and their parts.
3. In kindergarten, this expectation connects with the work of analyzing, comparing, creating, and composing shapes.
4. In future grades, students calculate area and surface area of these and other shapes.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

K.G.B. Geometry: Analyze, compare, create, and compose shapes.

Evidence Outcomes

Students Can:

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length). (CCSS: K.G.B.4)
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. (CCSS: K.G.B.5)
6. Compose simple shapes to form larger shapes. *For example, “Can you join these two triangles with full sides touching to make a rectangle?”* (CCSS: K.G.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use experiences with multiple examples of a type of shape to develop a concept image (see glossary) of that shape from which they can abstract common features. (MP2)
2. Model shapes in the world by building them with components or drawing representations of them. (MP4)
3. Use patterns or structures when making comparisons or compositions of shapes. (MP7)

Inquiry Questions:

1. Can you change a shape into a different kind of shape by rotating it?
2. What kinds of pictures can you make by combining shapes?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In preschool, students understand and use language related to directionality, order, and the position of objects, such as up/down and in front/behind.
3. In kindergarten, this expectation connects with identifying and describing shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
4. In Grade 1, students classify, compose, and partition shapes.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

1.NBT.A. Number & Operations in Base Ten: Extend the counting sequence.

Evidence Outcomes

Students Can:

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. (CCSS: 1.NBT.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make use of the base-ten counting structure when using special words at the decades, like “sixty” and “seventy.” (MP7)

Inquiry Questions:

1. When might someone want to count by tens instead of ones?
2. Which numbers can be written with two numerals and which numbers are written with three?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students count to 100 by ones and tens, count forward from a given number, and connect counting to cardinality.
3. In Grade 1, this expectation connects with understanding place value and with adding and subtracting within 20.
4. In Grade 2, students extend their place value understanding to hundreds and three-digit numbers, and use this along with the properties of operations to add and subtract within 1000 and fluently add and subtract within 100.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

1.NBT.B. Number & Operations in Base Ten: Understand place value.

Evidence Outcomes

Students Can:

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: (CCSS: 1.NBT.B.2)
    a. 10 can be thought of as a bundle of ten ones — called a “ten.” (CCSS: 1.NBT.B.2.a)
    b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS: 1.NBT.B.2.b)
    c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (CCSS: 1.NBT.B.2.c)
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. (CCSS: 1.NBT.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of quantities and their relationships in problem situations. (MP1)
2. Abstract 10 ones into a single conceptual object called a ten. (MP2)
3. Model ones and tens with objects and mathematical representations. (MP4)
4. See the structure of a number as its base-ten units. (MP7)

Inquiry Questions:

1. What does the position of a digit tell you about its value?
2. What are two ways to describe the number 30?
3. Why was a place value system developed? What might numbers look like without it?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students decompose numbers from 11 to 19 into ten ones and further ones.
3. In Grade 1, this expectation connects with extending the counting sequence and using place value understanding and properties of operations to add and subtract within 100.
4. In Grade 2, students understand hundreds and place value of three-digit numbers, and use this along with the properties of operations to add and subtract.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP7. Look for and make use of structure.

Grade Level Expectation:

1.NBT.C. Number & Operations in Base Ten: Use place value understanding and properties of operations to add and subtract.

Evidence Outcomes

Students Can:

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. (CCSS: 1.NBT.C.4)
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. (CCSS: 1.NBT.C.5)
6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS: 1.NBT.C.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Perform computation with addition and subtraction while making connections to the properties of operations and to place value structure. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Model quantities with drawings or equations to make sense of place value. (MP1)
3. Use the base-ten structure to add and subtract, including adding and subtracting multiples of ten. (MP7)

Inquiry Questions:

1. Can you add or subtract ten without having to count by ones?
2. How does modeling addition look different if you add tens and ones separately compared to counting on by tens then by ones?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students model and describe addition as putting together and adding to, and subtraction as taking part and taking from, using objects or drawings. Students also work with numbers 11–19 to gain foundations for place value.
3. In Grade 1, this expectation connects with understanding place value and adding and subtracting within 20.
4. In Grade 2, students understand place value for three-digit numbers and use that understanding and properties of operations to add and subtract within 1000 and fluently add and subtract within 100.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP4. Model with mathematics.

Grade Level Expectation:

1.OA.A. Operations & Algebraic Thinking: Represent and solve problems involving addition and subtraction.

Evidence Outcomes

Students Can:

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.A.1)
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS: 1.OA.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of problems by relating objects, drawings, and equations. (MP1)
2. Use cubes, number racks, ten frames and other models to represent addition and subtraction situations in real-world contexts. (MP4)

Inquiry Questions:

1. How can you use cubes to help you compare two numbers?
2. (Given a representation of a value less than ten) How many more do you need to make ten?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students add and subtract within 10 by using objects or drawings to represent problems.
3. In Grade 1, this expectation connects with comparing, adding, and subtracting numbers, including measurement and data activities.
4. In Grade 2, students represent and solve real-world problems involving addition and subtraction within 100, with fluency expected within 20.

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Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP7. Look for and make use of structure.

Grade Level Expectation:

1.OA.B. Operations & Algebraic Thinking: Understand and apply properties of operations and the relationship between addition and subtraction.

Evidence Outcomes

Students Can:

3. Apply properties of operations as strategies to add and subtract. (Students need not use formal terms for these properties.) *Examples: If* $8 + 3 = 11$ *is known, then* $3 + 8 = 11$ *is also known. (Commutative property of addition.) To add* $2 + 6 + 4$*, the second two numbers can be added to make a ten, so* $2 + 6 + 4 = 2 + 10 = 12$*. (Associative property of addition.)* (CCSS: 1.OA.B.3)
4. Understand subtraction as an unknown-addend problem. *For example, subtract* $10 - 8$ *by finding the number that makes* 10 *when added to* 8*.* (CCSS: 1.OA.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of addition and subtraction by applying properties of operations and working with different problem types (see Appendix, Table 1). (MP1)
2. Use properties of operations to recognize equivalent forms of equations. (MP7)

Inquiry Questions:

1. How could you explain why $3 + 8$ and $8 + 3$ both equal 11?
2. How can you use the number line to show how you might use adding OR subtracting to solve the same problem?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students model and describe addition as putting together and adding to, and subtraction as taking apart and taking from, using objects or drawings.
3. In Grade 1, this expectation connects with representing and solving problems involving addition and subtraction and with adding and subtracting within 20.
4. In future grades, students use place value understanding and properties of operations to add and subtract within larger number ranges, then to perform multi-digit arithmetic. Later, students use these concepts to build fractions from unit fractions, and to apply and extend their understandings of arithmetic to algebraic expressions.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

1.OA.C. Operations & Algebraic Thinking: Add and subtract within 20.

Evidence Outcomes

Students Can:

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (CCSS: 1.OA.C.5)
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$). (CCSS: 1.OA.C.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use multiple strategies to think about problems and see how the quantities involved support the use of some strategies over others. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make use of the structure of numbers when making tens or when creating equivalent but easier or known sums. (MP7)

Inquiry Questions:

1. Which would you prefer when adding $4 + 7$: starting with 7 and counting up 4 or starting with 4 and counting up 7? Why?
2. Why does knowing doubles like $4 + 4$ or $5 + 5$ help when adding $4 + 5$?
3. How does counting on to add and subtract within 20 make it easier to use fingers even though we have only 10 fingers?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students understand the relationship between numbers and quantities and connect counting to cardinality.
3. In Grade 1, this expectation connects with place value understanding, properties of addition and subtraction, the relationship between addition and subtraction, and with representing and solving problems involving addition and subtraction.
4. In Grade 2, students fluently add and subtract within 20 using mental strategies and know from memory all sums of two one-digit numbers.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

Grade Level Expectation:

1.OA.D. Operations & Algebraic Thinking: Work with addition and subtraction equations.

Evidence Outcomes

Students Can:

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?* $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$. (CCSS: 1.OA.D.7)
8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 + ? = 11$, $5 = \_ - 3$, $6 + 6 = \_$. (CCSS: 1.OA.D.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of quantities and their relationships in problem situations. (MP2)
2. Question assumptions about the meaning of the equals sign and construct viable arguments. (MP3)

Inquiry Questions:

1. What does it mean for two sides of an equation to be "equal"? How can $2 +$ 3 "equal" 5?
2. (Given $4 = 4$ If you add 2 more to the 4 on the right, how many do you need to add on the left to make a true statement? How would you write that as an equation?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students represent addition and subtraction with equations without needing to understand the meaning of the equal sign.
3. In Grade 1, this expectation connects with representing and solving problems involving addition and subtraction.
4. In Grade 2, students work with equal groups of objects to gain foundations for multiplication. In Grade 4, students build fractions from unit fractions and apply addition and subtraction to concepts of angle and angle measurement.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

1.MD.A. Measurement & Data: Measure lengths indirectly and by iterating length units.

Evidence Outcomes

Students Can:

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object. (CCSS: 1.MD.A.1)
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* (CCSS: 1.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Abstract comparisons between lengths using statements like $A > B$. (MP2)
2. Use the transitive property to explain if A is longer than B, and B is longer than C, then A must be longer than C. (MP3)
3. Devise different ways to represent the same data set and discuss the strengths and weaknesses of each representation. (MP5)
4. Consider the endpoints of objects when measuring and making comparisons. (MP6)

Inquiry Questions:

1. How is it possible for 5 sticks placed end-to-end to be equal in length to 6 sticks placed end-to-end?
2. Which is longer, the total length of two sticks placed end-to-end vertically or the same two sticks placed end-to-end horizontally?
3. What objects in this classroom are the same length as (or longer than, or shorter than) your forearm?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In kindergarten, students directly compare two objects with a measurable attribute in common.
3. In Grade 1, this expectation is part of a progression of learning that develops conceptions of comparison, conservation, seriation, and iteration.
4. In Grade 2, students measure and estimate lengths in standard units.

Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

1.MD.B. Measurement & Data: Tell and write time.

Evidence Outcomes

Students Can:

3. Tell and write time in hours and half-hours using analog and digital clocks. (CCSS: 1.MD.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Tell and manage time to be both personally responsible and responsible to the needs of others. (Personal Skills: Personal Responsibility)
2. Recognize that time is a quantity that can be measured with different degrees of precision. (MP6)

Inquiry Questions:

1. How long is two half-hours?
2. If the time is 2:30, where would the minute hand be pointing on an analog clock?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In kindergarten, students are not expected to learn how to tell and write time.
3. In Grade 2, students tell and write time from analog and digital clocks to the nearest five minutes.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

1.MD.C. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (CCSS: 1.MD.C.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Ask and answer questions about categorical data based on representations of the data. (MP1)
2. Group similar individual objects together and abstract those objects into a new conceptual group. (MP2)
3. Devise different ways to display the same data set then discuss relative strengths and weaknesses of each scheme. (MP5)
4. Use appropriate labels and units of measure. (MP6)

Inquiry Questions:

1. How do different representations of data indicate there are more objects in one category than in another category?
2. How can objects be categorized in different ways?
3. How can an object's attributes determine if it does not belong with other objects in a group?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In kindergarten, students classify objects into given categories, count the numbers of objects in each category, and sort the categories by count.
3. In Grade 1, this expectation supports representing and solving problems involving addition and subtraction, which is major work of the grade.
4. In Grade 2, students draw a picture graph and a bar graph to represent a data set with up to four categories, and solve put-together, take-apart, and compare problems using the information in a bar graph.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

1.G.A. Geometry: Reason with shapes and their attributes.

Evidence Outcomes

Students Can:

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. (CCSS: 1.G.A.1)
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names, such as “right rectangular prisms.”) (CCSS: 1.G.A.2)
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. (CCSS: 1.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Demonstrate flexibility, imagination, and inventiveness in composing two-dimensional and three-dimensional shapes to create composite shapes. (Entrepreneurial Skills: Informed Risk Taking)
2. Sort, classify, build, or draw shapes in terms of defining attributes versus non-defining attributes. (MP1)
3. Determine how to partition a given circle or rectangle into two and four equal shares and describe the whole in terms of equal shares. (MP2)
4. Justify whether a shape belongs in a given category by differentiating between defining attributes and non-defining attributes. (MP3)
5. Analyze how composite shapes can be formed by, or decomposed into, basic shapes. (MP7)

Inquiry Questions:

1. Which properties of shapes are most important when you decide if a shape belongs in a group with other shapes?
2. What kinds of objects can you find in your school or home that are made up of two or more different shapes being put together?
3. In how many different ways can you create two or four equal shares in a rectangle?

Coherence Connections:

1. This expectation is an addition to the major work of the grade.
2. In kindergarten, students identify, describe, analyze, compare, create, and compose shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
3. In Grade 2, students recognize and draw shapes having specified attributes and partition circles and rectangles into two, three, or four equal shares. In Grade 3, students develop understanding of fractions as numbers.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make use of structure.

Grade Level Expectation:

2.NBT.A. Number & Operations in Base Ten: Understand place value.

Evidence Outcomes

Students Can:

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: (CCSS: 2.NBT.A.1)
   a. 100 can be thought of as a bundle of ten tens — called a "hundred." (CCSS: 2.NBT.A.1.a)
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (CCSS: 2.NBT.A.1.b)
2. Count within 1000; skip-count by 5s, 10s, and 100s. (CCSS: 2.NBT.A.2)
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS: 2.NBT.A.3)
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. (CCSS: 2.NBT.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Abstract 10 ones into a single conceptual object called a ten and abstract 100 ones or 10 tens into a single conceptual object called a hundred. (MP2)
2. Compose, decompose, and compare three-digit numbers according to their base-ten structure. (MP7)

Inquiry Questions:

1. How many hundreds are in the number "four hundred five"? How do you know? How many tens are in the number "four hundred five? How do you know?
2. How many times do you need to skip count by 5s to count as far as skip counting by 10s once?
3. How many times do you need to skip count by 10s to count as far as skip counting by 100 once?
4. Why is any two-digit number that starts with 5 always larger than a two-digit number that starts with 3?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students understand place value for two-digit numbers.
3. In Grade 2, this expectation connects with using place value understanding and properties of operations to add and subtract and with working with equal groups of objects to gain foundations for multiplication.
4. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP7. Look for and make use of structure.

Grade Level Expectation:

2.NBT.B. Number & Operations in Base Ten: Use place value understanding and properties of operations to add and subtract.

Evidence Outcomes

Students Can:

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 2.NBT.B.5)
6. Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS: 2.NBT.B.6)
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS: 2.NBT.B.7)
8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS: 2.NBT.B.8)
9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.) (CCSS: 2.NBT.B.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Relate concrete or mental strategies for adding and subtracting within 100 to a written method. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of place value by modeling quantities with drawings or equations. (MP1)
3. Use the base-ten structure to add and subtract, composing and decomposing ones, tens, and hundreds as necessary. (MP7)

Inquiry Questions:

1. Why might it be helpful to view subtraction as an unknown addend problem? (e.g., $278 + ? = 425$)
2. How might you rewrite $38 + 47 + 93 + 62$ to make it easier to solve? How do you know it is OK to rewrite it?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students use place value and properties of operations to make sense of the relationship between addition and subtraction.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction and fluently adding and subtracting within 20.
4. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic, including fluently adding and subtracting within 1000.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

2.OA.A. Operations & Algebraic Thinking: Represent and solve problems involving addition and subtraction.

Evidence Outcomes

Students Can:

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (see Appendix, Table 1) (CCSS: 2.OA.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Decontextualize word problems, use mathematics to solve, and then recontextualize to provide the answer in context. (MP2)
2. Represent situations in word problems using drawings and equations with symbols for unknown numbers. (MP4)

Inquiry Questions:

1. (Given a word problem) What is the unknown quantity in this problem?
2. (Given an addition or subtraction problem) How might you use a model to represent this problem?
3. Does the word "more" in a word problem always mean that you will use addition to solve the problem? Why or why not?

Coherence Connections:

1. This expectation represents the major work of the grade.
2. In Grade 1, students use place value understanding and properties of operations to represent and solve problems involving addition and subtraction.
3. This expectation connects with other ideas in Grade 2: (a) using place value understanding and properties of operations to add and subtract, (b) relating addition and subtraction to length, (c) working with time and money, and (d) representing and interpreting data.
4. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.

Prepared Graduates:

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

Grade Level Expectation:

2.OA.B. Operations & Algebraic Thinking: Add and subtract within 20.

Evidence Outcomes

Students Can:

2. Fluently add and subtract within 20 using mental strategies. (See 1.OA.C.6 for a list of strategies.) By end of Grade 2, know from memory all sums of two one-digit numbers. (CCSS: 2.OA.B.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize those problems that can be solved mentally versus those that require the use of objects, diagrams, or equations. (MP5)
2. Add and subtract within 20 quickly, accurately, and flexibly. (MP6)

Inquiry Questions:

1. How can you use addition and subtraction facts you know to quickly determine facts that you don't know?
2. Why do you think it is important to know your addition and subtraction facts?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students use objects and drawings to add and subtract within 20 in preparation for fluency with mental strategies in Grade 2.
3. In Grade 2, this expectation connects with using place value understanding and properties of operations to add and subtract within 1000 and fluently add and subtract within 100.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

Grade Level Expectation:

2.OA.C. Operations & Algebraic Thinking: Work with equal groups of objects to gain foundations for multiplication.

Evidence Outcomes

Students Can:

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. (CCSS: 2.OA.C.3)
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (CCSS: 2.OA.C.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explore the arrangement of objects and how some arrangements afford mathematical power to solve problems. (Entrepreneurial Skills: Creativity/Innovation)
2. Reason about what it means for numbers to be even and odd. (MP2)
3. Explain why a group of objects is even or odd and if a strategy for deciding works with any group of objects. (MP3)

Inquiry Questions:

1. What does it mean for a number to be even?
2. Do two equal addends always result in an even sum? Why or why not?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 1, students work with addition and subtraction equations.
3. In Grade 2, this expectation connects with understanding place value for three-digit numbers.
4. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

2.MD.A. Measurement & Data: Measure and estimate lengths in standard units.

Evidence Outcomes

Students Can:

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (CCSS: 2.MD.A.1)
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (CCSS: 2.MD.A.2)
3. Estimate lengths using units of inches, feet, centimeters, and meters. (CCSS: 2.MD.A.3)
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS: 2.MD.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Consider the correctness of another students' measurement in which they lined up three large and four small blocks and claimed a path was "seven blocks long." (MP3)
2. Choose between different measurement tools depending on the objects they need to measure. (MP5)
3. Determine when it is appropriate to estimate an object's length or when a more precise measurement is needed. (MP6)

Inquiry Questions:

1. What do the numbers on a ruler represent?
2. What is the more appropriate tool for measuring the length of your school hallway, a 1-foot ruler or a 25-foot measuring tape?
3. When is it appropriate to estimate length? When is it not appropriate?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students measure lengths indirectly and by iterating length units.
3. In Grade 2, this expectation connects with relating addition and subtraction to length and with representing and interpreting data.
4. In Grade 3, students (a) develop understanding of fractions as numbers, (b) solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects, and (c) use concepts of area and relate area to multiplication and to addition.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

2.MD.B. Measurement & Data: Relate addition and subtraction to length.

Evidence Outcomes

Students Can:

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (CCSS: 2.MD.B.5)
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1,2, …, and represent whole-number sums and differences within 100 on a number line diagram. (CCSS: 2.MD.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize problems involving lengths and identify possible solutions. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Build on experiences with measurement tools to understand number lines as a more abstract tool for working with quantities. (MP2)
3. Use mathematical representations, like drawings and equations, to model scenarios described in word problems. (MP4)

Inquiry Questions:

1. When might it be necessary to measure parts of objects and then combine those parts together?
2. How is a number line like a ruler?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 1, students add and subtract within 20 and express the length of an object as a whole number of length units.
3. In Grade 2, this expectation connects with measuring and estimating lengths in standard units and with representing and interpreting data.
4. In Grade 3, students develop an understanding of a fraction as a number on a number line.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP6. Attend to precision.

Grade Level Expectation:

2.MD.C. Measurement & Data: Work with time and money.

Evidence Outcomes

Students Can:

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS: 2.MD.C.7)
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have two dimes and three pennies, how many cents do you have?* (CCSS: 2.MD.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Tell and manage time to be both personally responsible and responsible to the needs of others. (Personal Skills: Personal Responsibility)
2. Make sense of word problems involving money. (MP1)
3. Recognize that time is a quantity that can be measured with different degrees of precision. (MP6)

Inquiry Questions:

1. If the time is 2:25, where would the minute hand be pointing on an analog clock?
2. Does the size of a coin indicate the value of the coin?
3. How is money like our base-ten number system, where it takes ten of one unit to make the next unit (ten ones makes a ten, ten tens make a hundred)? In what ways is it different?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 1, students tell and write time in hours and half-hours using analog and digital clocks.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction.
4. In Grade 3, students tell and write time to the nearest minute and measure time intervals in minutes.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

2.MD.D. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS: 2.MD.D.9)
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems (see Appendix, Table 1) using information presented in a bar graph. (CCSS: 2.MD.D.10)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Organize objects according to measures or categories to help make sense of problems. (MP1)
2. Organize measurement and categorical data into categories based on size or type so comparisons can be made between categories instead of between individual objects. (MP2)
3. Discuss ways in which bar graph orientation (horizontal or vertical), order, thickness, spacing, shading, colors, etc. make the graphs easier or more difficult to interpret. (MP5)

Inquiry Questions:

1. How is organizing objects by length measurements, rounded to the nearest unit, similar to and different from organizing objects by categories?
2. (Given a bar graph representation of up to four categories of animals) How many more birds are there than hippos? How many more giraffes would there need to be in order for the number of giraffes to equal the number of elephants?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 1, students organize, represent, and interpret data with up to three categories and compare how many more or less are in one category than another.
3. In Grade 2, this expectation connects with representing and solving problems involving addition and subtraction and with relating addition and subtraction to length.
4. In Grade 3, students draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.

Grade Level Expectation:

2.G.A. Geometry: Reason with shapes and their attributes.

Evidence Outcomes

Students Can:

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (CCSS: 2.G.A.1)
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS: 2.G.A.2)
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. (CCSS: 2.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Demonstrate flexibility, imagination, and inventiveness in drawing shapes having specified attributes and in partitioning circles and rectangles into equal shares. (Entrepreneurial Skills: Informed Risk Taking)
2. Explore various ways of partitioning shapes into equal shares, such as different methods for dividing a square into fourths, to understand that each partition, regardless of shape, represents an equal share of the square. (MP2)
3. Engage in spatial structuring by tiling rectangles with rows and columns of squares to build understanding of two-dimensional regions. (MP7)

Inquiry Questions:

1. How many different triangles can you draw where two of the sides have the same length?
2. (Given a rectangle) Can you divide this rectangle into three equal parts in more than one way?

Coherence Connections:

1. This expectation is in addition to the major work of Grade 2.
2. In Grade 1, students reason with shapes and their attributes, distinguish between defining and non-defining attributes, compose two-dimensional shapes, and partition circles and rectangles into halves and fourths.
3. In Grade 3, students develop understanding of fractions as numbers, use concepts of area and relate area to multiplication and to addition, and understand that shared attributes in different categories of shapes can define a larger category.

Prepared Graduates:

MP6. Attend to precision.

MP7. Look for and make use of structure.

Grade Level Expectation:

3.NBT.A. Number & Operations in Base Ten: Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of algorithms may be used.

Evidence Outcomes

Students Can:

1. Use place value understanding to round whole numbers to the nearest 10 or 100. (CCSS: 3.NBT.A.1)
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 3.NBT.A.2)
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations. (CCSS: 3.NBT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly exhibit understanding of a variety of strategies when performing multi-digit arithmetic. (Personal Skills: Adaptability/Flexibility)
2. Demonstrate place value understanding by precisely referring to digits according to their place value. (MP6)
3. Recognize and use place value and properties of operations to structure algorithms and other representations of multi-digit arithmetic. (MP7)

Inquiry Questions:

1. How is rounding whole numbers to the nearest 10 or 100 useful?
2. Do different strategies for solving lead to different answers when we add or subtract? Why or why not?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 2, students use place value understanding and properties of operations to add and subtract fluently within 100.
3. This expectation connects to other ideas in Grade 3: (a) an understanding of multiplication, (b) knowing the relationship between multiplication and division, and (c) the concept of area and its relationship to multiplication and division.
4. In Grade 4, students generalize place value understanding for multi-digit whole numbers and use that understanding and the properties of operations to perform multi-digit arithmetic.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.NF.A. Number & Operations—Fractions: Develop understanding of fractions as numbers.

Evidence Outcomes

Students Can:

1. Describe a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$. (CCSS: 3.NF.A.1)
2. Describe a fraction as a number on the number line; represent fractions on a number line diagram. (CCSS: 3.NF.A.2)
   a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. (CCSS: 3.NF.A.2.a)
   b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. (CCSS: 3.NF.A.2.b)
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (CCSS: 3.NF.A.3)
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (CCSS: 3.NF.A.3.a)
   b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2}=\frac{2}{4}$, $\frac{4}{6}=\frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. (CCSS: 3.NF.A.3.b)
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express* 3 *in the form* $3=\frac{3}{1}$*; recognize that* $\frac{6}{1}=6$*; locate* $\frac{4}{4}$ *and* 1 *at the same point of a number line diagram.* (CCSS: 3.NF.A.3.c)
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. (CCSS: 3.NF.A.3.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly describe fractions both as parts of other numbers but also as numbers themselves. (Personal Skills: Adaptability/Flexibility)
2. Analyze and use information presented visually (for example, number lines, fraction models, and diagrams representing parts and wholes) that support an understanding of fractions as numbers. (Entrepreneurial Skills: Literacy/Reading)
3. Reason about the number line in a new way by understanding and using fractional parts between whole numbers. (MP2)
4. Critique the reasoning of others when comparing fractions that may refer to different wholes. (MP3)
5. Use the structure of fractions to locate and compare fractions on a number line. (MP7)

Inquiry Questions:

1. How does the denominator of a unit fraction connect to the number of unit fractions that must be added to make a whole?
2. When the numerators of two different fractions are the same, how can the denominators be used to compare them?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students (a) relate addition and subtraction to length, (b) measure and estimate lengths in standard units, and (c) reason with shapes and their attributes, including partitioning circles and rectangles into halves, thirds, and fourths.
3. In Grade 3, this expectation connects to the solving of problems involving measurement and estimation of intervals of time, liquid volumes, and mass of objects and is further supported by the expectation to represent and interpret data.
4. In Grade 4, students build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers and extend their understanding of fraction equivalence and ordering. In Grade 6, students apply and extend previous understandings of numbers (including fractions) to the system of rational numbers.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

3.OA.A. Operations & Algebraic Thinking: Represent and solve problems involving multiplication and division.

Evidence Outcomes

Students Can:

1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7. (CCSS: 3.OA.A.1)
2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$. (CCSS: 3.OA.A.2)
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (see Appendix, Table 2) (CCSS: 3.OA.A.3)
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \_ \div 3$, $6 \times 6 = ?$ (CCSS: 3.OA.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving multiples and parts using multiplication and division. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of missing numbers in equations by using the relationship between multiplication and division. (MP1)
3. Reason abstractly about numbers of groups and the size of groups to make meaning of the quantities involved in multiplication and division. (MP2)
4. Use arrays to represent whole-number multiplication and division problems. (MP4)

Inquiry Questions:

1. How can an array be decomposed in a way that connects it to known multiplication facts? How can arrays be used to write and solve multiplication problems?
2. How can the area and one side of a rectangle be used to write and solve a division problem?
3. How could the number of dots in an array be counted without counting them one by one?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. In Grade 3, this expectation connects to understanding properties of multiplication, the relationship between multiplication and division, and to fluently multiplying and dividing within 100.
4. In Grade 4, students (a) use the four operations with whole numbers to solve problems, (b) build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers, and (c) solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. In Grade 5, students apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.OA.B. Operations & Algebraic Thinking: Apply properties of multiplication and the relationship between multiplication and division.

Evidence Outcomes

Students Can:

5. Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If* $6 \times 4 = 24$ *is known, then* $4 \times 6 = 24$ *is also known. (Commutative property of multiplication.)* $3 \times 5 \times 2$ *can be found by* $3 \times 5 = 15$, *then* $15 \times 2 = 30$, *or by* $5 \times 2 = 10$, *then* $3 \times 10 = 30$. *(Associative property of multiplication.) Knowing that* $8 \times 5 = 40$ *and* $8 \times 2 = 16$, *one can find* 8×7 *as* $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. *(Distributive property.)* (CCSS: 3.OA.B.5)
6. Interpret division as an unknown-factor problem. *For example, find* $32 \div 8$ *by finding the number that makes* 32 *when multiplied by* 8. (CCSS: 3.OA.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly work with different but related arrangements of factors and products or dividends, divisors, and quotients. (Personal Skills: Adaptability/Flexibility)
2. Use properties of operations to argue for or against the equivalence of different expressions. (MP3)
3. Be specific with explanations and symbols when describing operations using multiplication and division. (MP6)
4. Use the relationship between multiplication and division to rewrite division problems as multiplication. (MP7)

Inquiry Questions:

1. What are all of the equations that can be written to represent the relationship between the area of a (specific) rectangle and its side lengths?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. This expectation connects to other ideas in Grade 3: (a) multiplication and division within 100, (b) solving problems involving the four operations and identifying and explaining patterns in arithmetic, (c) understanding properties of multiplication and the relationship between multiplication and division, and (d) understanding concepts of area and the relationship to multiplication and division.
4. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

3.OA.C. Operations & Algebraic Thinking: Multiply and divide within 100.

Evidence Outcomes

Students Can:

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. (CCSS: 3.OA.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Efficiently solve multiplication and division problems by using facts committed to memory. (Professional Skills: Task/Time Management)
2. Recognize the relationship between skip counting and the solutions to problems involving multiplication and division. (MP7)

Inquiry Questions:

1. How can I use multiplication facts that I know to solve multiplication problems I do not yet know? (for example, using $5 \times 4 + 2 \times 4$ to solve 7×4)?
2. How can I use models and strategies to show what I know about multiplication?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. In Grade 3, this expectation connects with representing and solving problems involving the four operations.
4. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic, solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and gain familiarity with factors and multiples.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP6. Attend to precision.

Grade Level Expectation:

3.OA.D. Operations & Algebraic Thinking: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Evidence Outcomes

Students Can:

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This evidence outcome is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order of operations when there are no parentheses to specify a particular order.) (CCSS: 3.OA.D.8)
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* (CCSS: 3.OA.D.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving the four operations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Explain patterns in arithmetic. (MP3)
3. Mathematically model changes in quantities described in real-world contexts using the appropriate numbers, operations, symbols, and letters to represent unknowns. (MP4)
4. Complement arithmetic strategies with mental computation and estimation to assess answers for accuracy. (MP6)

Inquiry Questions:

1. How can a visual model support making sense of and solving word problems?
2. How can the patterns in addition and/or multiplication tables help predict probable solutions to a given problem?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students represent and solve one- and two-step word problems involving addition and subtraction.
3. This expectation connects to several ideas in Grade 3: (a) representing and solving problems involving multiplication and division, (b) multiplying and dividing within 100, (c) solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects, and (d) using concepts of area and relating area to multiplication and to addition.
4. In Grade 4, students use the four operations with whole numbers to solve problems.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

3.MD.A. Measurement & Data: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Evidence Outcomes

Students Can:

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. (CCSS: 3.MD.A.1)
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (This excludes compound units such as cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (This excludes multiplicative comparison problems, such as problems involving notions of “times as much.” See Appendix, Table 2.) (CCSS: 3.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use units of measurement appropriate to the type and magnitude of the quantity being measured. (Professional Skills: Information Literacy)
2. Make sense of problems involving measurement by building on real-world knowledge of time and objects and an understanding of the relative sizes of units. (MP1)
3. Represent problems of time and measurement with equations, drawings, or diagrams. (MP4)
4. Use appropriate measures and measurement instruments for the quantities given in a problem. (MP5)

Inquiry Questions:

1. How can elapsed time be modeled on a number line to support the connection to addition and subtraction?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units and work with time and money.
3. In Grade 3, this expectation connects to developing an understanding of fractions as numbers, solving problems involving the four operations, and identifying and explaining patterns in arithmetic.
4. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

3.MD.B. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent* 5 *pets.* (CCSS: 3.MD.B.3)
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (CCSS: 3.MD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze data to distinguish the factual evidence offered, to reason about judgments, to draw conclusions, and to speculate about ideas the data represents. (Entrepreneurial Skills: Literacy/Reading)
2. Abstract real-world quantities into scaled graphs. (MP2)
3. Model real-world quantities with statistical representations such as bar graphs and line graphs. (MP4)

Inquiry Questions:

1. How can working with pictures and bar graphs connect mathematics to the world around us?
2. How does changing the scale of a bar graph or line plot change the appearance of the data?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 2, students represent and interpret length by measuring objects, make line plots, and use picture and bar graphs to represent categorical data.
3. In Grade 3, this expectation supports developing an understanding of fractions as numbers.
4. In Grade 4, students represent and interpret data by making line plots representing fractional measurements and solving addition and subtraction problems using information presented in line plots.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.MD.C. Measurement & Data: Geometric measurement: Use concepts of area and relate area to multiplication and to addition.

Evidence Outcomes

Students Can:

5. Recognize area as an attribute of plane figures and understand concepts of area measurement. (CCSS: 3.MD.C.5)
   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.C.5.a)
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. (CCSS: 3.MD.C.5.b)
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). (CCSS: 3.MD.C.6)
7. Use concepts of area and relate area to the operations of multiplication and addition. (CCSS: 3.MD.C.7)
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 3.MD.C.7.a)
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (CCSS: 3.MD.C.7.b)
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (CCSS: 3.MD.C.7.c)
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. (CCSS: 3.MD.C.7.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Defend calculations of area using multiplication and by tiling the area with square units and comparing the results. (MP3)
2. Understand how to use a one-dimensional measurement tool, like a ruler, to make two-dimensional measurements of area. (MP5)
3. Be precise by describing area in square rather than linear units. (MP6)
4. Use areas of rectangles to exhibit the structure of the distributive property. (MP7)

Inquiry Questions:

1. Given three pictures of different rectangles with unknown dimensions, how can you determine which rectangle covers the most area?
2. How does computing the area of a rectangle relate to closed arrays?
3. How can the area of an E-shaped or H-shaped figure be calculated?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units and reason with shapes and their attributes.
3. This expectation connects to other ideas in Grade 3: (a) recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures, (b) applying properties of multiplication and the relationship between multiplication and division, and (c) solving problems involving the four operations and identifying and explaining patterns in arithmetic.
4. In Grade 4, students solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit. In Grade 5, students relate volume to multiplication and to addition and also extend previous understandings of multiplication and division to multiply and divide fractions.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.

Grade Level Expectation:

3.MD.D. Measurement & Data: Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Evidence Outcomes

Students Can:

8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS: 3.MD.D.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of the relationship between area and perimeter by calculating both for rectangles of varying sizes and dimensions. (MP1)
2. Model perimeters of objects in the world with polygons and the sum of their side lengths. (MP4)

Inquiry Questions:

1. What are all the pairs of side lengths that can create a rectangle with the same area, such as 12 square units?
2. Is it possible for two rectangles to have the same area but different perimeters?
3. Is it possible for two rectangles to have the same perimeter but different areas?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units.
3. In Grade 3, this expectation connects to understanding concepts of area, relating area to multiplication and to addition, and solving problems involving the four operations.
4. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP7. Look for and make use of structure.

Grade Level Expectation:

3.G.A. Geometry: Reason with shapes and their attributes.

Evidence Outcomes

Students Can:

1. Explain that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (CCSS: 3.G.A.1)
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into* 4 *parts with equal area, and describe the area of each part as* $\frac{1}{4}$ *of the area of the shape.* (CCSS: 3.G.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Work with others to name and categorize shapes. (Civic/Interpersonal Skills: Collaboration/Teamwork)
2. Analyze, compare, and use the properties of geometric shapes to classify them into abstracted categories and describe the similarities and differences between those categories. (MP2)
3. Convince others or critique their reasoning when deciding if a shape belongs to certain categories of polygons. (MP3)
4. Decompose geometric shapes into polygons of equal area. (MP7)

Inquiry Questions:

1. Can you draw a quadrilateral that is not a rhombus, rectangle, or square?
2. (Given two identical squares) Divide each of these squares into four equal parts, but in different ways. If you compare a part of one with a part of the other, are their areas the same? How do you know?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 2, students reason with shapes and their attributes.
3. In Grade 3, this expectation connects to developing an understanding of fractions as numbers.
4. In Grade 4, students draw and identify lines and angles and also classify shapes by properties of their lines and angles.

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Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

4.NBT.A. Number & Operations in Base Ten: Generalize place value understanding for multi-digit whole numbers.

Evidence Outcomes

Students Can:

1. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that* $700 \div 70 = 10$ *by applying concepts of place value and division.* (CCSS: 4.NBT.A.1)
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. (CCSS: 4.NBT.A.2)
3. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Write multi-digit whole numbers in different forms to support claims and justify reasoning. (Entrepreneurial Skills: Literacy/Writing)
2. Use the structure of the base-ten number system to read, write, compare, and round multi-digit numbers. (MP7)

Inquiry Questions:

1. How do base ten area pieces or representations help with understanding multiplying by 10 or a multiple of 10? How can base ten area pieces be used to represent multiplying by 10 or a multiple of 10?
2. Imagine two four-digit numbers written on paper and some of the digits were smeared. If you saw just 325 ■ and 331 ■, could you determine which number was larger?
3. When is it helpful to use a rounded number instead of the exact number?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic.
3. In Grade 4, this expectation connects to using the four operations with multi-digit whole numbers to solve measurement and other problems.
4. In Grade 5, students extend their understanding of place value to decimals, and read, write, and compare decimals to thousandths.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

MP7. Look for and make use of structure.

Grade Level Expectation:

4.NBT.B. Number & Operations in Base Ten: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Evidence Outcomes

Students Can:

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. (CCSS: 4.NBT.B.4)
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.B.5)
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve multi-digit arithmetic problems. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Explain the process and result of multi-digit arithmetic. (MP3)
3. Precisely and efficiently add and subtract multi-digit numbers. (MP6)
4. Use the structure of place value to support the organization of mental and written multi-digit arithmetic strategies. (MP7)

Inquiry Questions:

1. How can a visual model be used to demonstrate the relationship between multiplication and division?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students use place value understanding and properties of operations to add and subtract within 1000 and to multiply and divide within 100.
3. In Grade 4, this expectation connects to using the four operations with whole numbers to solve problems.
4. In Grade 5, students understand the place value of decimals and perform operations with multi-digit whole numbers and with decimals to hundredths.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.NF.A. Number & Operations—Fractions: Extend understanding of fraction equivalence and ordering.

Evidence Outcomes

Students Can:

1. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (CCSS: 4.NF.A.1)
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. (CCSS: 4.NF.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explain the equivalence of fractions. (MP3)
2. Use visual models and benchmark fractions as tools to aid in fraction comparison. (MP5)
3. Precisely refer to numerators, denominators, parts, and wholes when explaining fraction equivalence and comparing fractions. (MP6)
4. Use 1, the multiplicative identity, to create equivalent fractions by structuring 1 in the fraction form $\frac{n}{n}$. (MP7)

Inquiry Questions:

1. Why does it work to compare fractions either by finding common numerators or by finding common denominators?
2. How can you be sure that multiplying a fraction by $\frac{n}{n}$ does not change the fraction's value?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students develop an understanding of fractions as numbers and the meaning of the denominator of a unit fraction.
3. In Grade 4, this expectation connects to building fractions from unit fractions, using decimal notation and comparing decimal fractions, and using the four operations with whole numbers to solve problems.
4. In Grade 5, students use equivalent fractions as a strategy to add and subtract fractions with unlike denominators and apply and extend previous understandings of multiplication and division to fractions.

Prepared Graduates:

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.NF.B. Number & Operations—Fractions: Build fractions from unit fractions.

Evidence Outcomes

Students Can:

3. Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. (CCSS: 4.NF.B.3)
    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS: 4.NF.B.3.a)
    b. Decompose a fraction into a sum of fractions with like denominators in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$. (CCSS: 4.NF.B.3.b)
    c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS: 4.NF.B.3.c)
    d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (CCSS: 4.NF.B.3.d)
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS: 4.NF.B.4)
    a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent $\frac{5}{4}$ as the product $5 \times \frac{1}{4}$, recording the conclusion by the equation $\frac{5}{4} = 5 \times \frac{1}{4}$.* (CCSS: 4.NF.B.4.a)
    b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times \frac{2}{5}$ as $6 \times \frac{1}{5}$, recognizing this product as $\frac{6}{5}$. (In general, $n \times \frac{a}{b} = \frac{n \times a}{b}$.)* (CCSS: 4.NF.B.4.b)
    c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (CCSS: 4.NF.B.4.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use the structure of fractions to perform operations with fractions and to understand and explain how the operations connect to the structure of fractions. (MP7)
2. Recognize the mathematical connections between the indicated operations with fractions and the corresponding operations with whole numbers. (MP8)

Inquiry Questions:

1. How is the addition of unit fractions similar to counting whole numbers?
2. How does multiplying two whole numbers relate to multiplying a fraction by a whole number?
3. (Given two fractions with like denominators, each of which is less than $\frac{1}{2}$) Before adding these two fractions, can you predict whether the sum will be greater than or less than 1? How do you know?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students develop understanding of fractions as numbers and represent and solve problems involving multiplication and division.
3. This expectation connects to other ideas in Grade 4: (a) using decimal notation for fractions and comparing decimal fractions, (b) using the four operations with whole numbers to solve problems, (c) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and (d) representing and interpreting data.
4. In Grade 5, students use equivalent fractions as a strategy to add and subtract fractions with unlike denominators and apply and extend previous understandings of multiplication to decimals.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.NF.C. Number & Operations—Fractions: Use decimal notation for fractions, and compare decimal fractions.

Evidence Outcomes

Students Can:

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) *For example, express* $\frac{3}{10}$ *as* $\frac{30}{100}$*, and add* $\frac{3}{10}+\frac{4}{100}=\frac{34}{100}$. (CCSS: 4.NF.C.5)
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite* 0.62 *as* $\frac{62}{100}$*; describe a length as* 0.62 *meters; locate* 0.62 *on a number line diagram.* (CCSS: 4.NF.C.6)
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model. (CCSS: 4.NF.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Approach adding, subtracting, and comparing problems with fractions and decimal fractions by reasoning about their values before or instead of applying an algorithm. (MP1)
2. Draw fraction models to reason about and compute with decimal fractions. (MP5)
3. Make use of the structure of place value to express and compare decimal numbers in tenths and hundredths. (MP7)

Inquiry Questions:

1. How does a fraction with a denominator of 10 or 100 relate to its decimal quantity?
2. How can visual models help to compare two decimal quantities?
3. How is locating a decimal on a number line similar to locating a fraction on a number line?

Coherence Connections:

1. This expectation represents major work of the grade.
2. This expectation connects to several ideas in Grade 4: (a) extending understanding of fraction equivalence and ordering, (b) building fractions from unit fractions, and (c) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
3. In Grade 5, students understand the decimal place value system and use it with the four operations.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.OA.A. Operations & Algebraic Thinking: Use the four operations with whole numbers to solve problems.

Evidence Outcomes

Students Can:

1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.A.1)
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Appendix, Table 2) (CCSS: 4.OA.A.2)
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of multi-step word problems by understanding the relationships between known and unknown quantities. (MP1)
2. Reason quantitatively with word problems by considering the units involved and how the quantities they describe increase or decrease with addition and subtraction or scale with multiplication and division. (MP2)
3. Use mathematics to model real-world problems requiring operations with whole numbers and contextually interpret remainders when they arise. (MP4)
4. Look for structures of commutativity and inverses of operations in solving whole number problems with the four operations. (MP7)

Inquiry Questions:

1. What makes a multiplicative comparison different from an additive comparison?
2. How can you recognize whether a comparison is multiplicative or additive?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students represent and solve problems involving multiplication and division, apply properties of multiplication and the relationship between multiplication and division, solve problems involving the four operations, and identify and explain patterns in arithmetic.
3. This expectation connects to other ideas in Grade 4: (a) using place value understanding and properties of operations to perform multi-digit arithmetic, (b) extending understanding of fraction equivalence and ordering, (c) building fractions from unit fractions, and (d) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4. In Grade 5, students apply and extend previous understandings of multiplication and division to multiply and divide fractions by fractions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.OA.B. Operations & Algebraic Thinking: Gain familiarity with factors and multiples.

Evidence Outcomes

Students Can:

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS: 4.OA.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason quantitatively to recognize that a number is a multiple of each of its factors. (MP2)
2. Use the relationship between factors and multiples for whole numbers. (MP7)
3. Look for, identify, and explain the regularities in determining whether a given number is a multiple of a given one-digit number and in determining if a given number is prime or composite. (MP8)

Inquiry Questions:

1. How can you use arrays to explore and determine all of the factors of a given number?
2. How are multiples and factors helpful in solving problems related to fractional parts of a whole number, such as $\frac{3}{5}$ of 20?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 3, students multiply and divide within 100.
3. In Grade 6, students compute fluently with multi-digit numbers, find common factors and multiples, and extend previous understandings of arithmetic to algebraic expressions.

Prepared Graduates:

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.OA.C. Operations & Algebraic Thinking: Generate and analyze patterns.

Evidence Outcomes

Students Can:

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* (CCSS: 4.OA.C.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explore and generate sequences of numbers or shapes that can be described mathematically. (Entrepreneurial Skills: Creativity/Innovation)
2. Notice when calculations are repeated and describe patterns in generalized, mathematical ways. (MP8)

Inquiry Questions:

1. If you were given a rule to add 4 to a starting number then to each number that follows, can you generate a sequence of odd numbers? How?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.
3. In Grade 5, students analyze pairs of patterns created from two given rules and describe and graph the corresponding relationships.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

4.MD.A. Measurement & Data: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Evidence Outcomes

Students Can:

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1,12), (2,24), (3,36), …* (CCSS: 4.MD.A.1)
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS: 4.MD.A.2)
3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* (CCSS: 4.MD.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Define quantities in measurement problems with both their magnitude and unit. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of quantities, their units, and their relationships in problem solving situations. (MP2)
3. Model real-world problems involving area and perimeter with equations, diagrams, and formulas, and use them to solve problems. (MP4)
4. Generate and use conversion tables to aid in measurement conversions, and represent measurement quantities on scaled line diagrams. (MP5)

Inquiry Questions:

1. How can you use what you know about place value to convert between km, m and cm? Does this also work for measurement of time (s, m, h)? Why or why not?
2. How can visual models help to make sense of measurement problems and intervals of time?
3. How many liters of juice are needed to fill 35 cups of 225 ml each?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 3, students solve problems involving multiplication and division and solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3. In Grade 4, this expectation connects to building fractions from unit fractions, using decimal notation for fractions, comparing decimal fractions, and using the four operations with whole numbers to solve problems.
4. In Grade 5, students apply and extend previous understandings of multiplication and division, convert like measurement units within a given measurement system, and relate volume to multiplication and to addition.

COLORADO
Department of Education

Prepared Graduates:

MP5. Use appropriate tools strategically.

Grade Level Expectation:

4.MD.B. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

4. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* (CCSS: 4.MD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Read and represent measurements recorded on line plots. (Professional Skills: Information Literacy)
2. Use a line plot to represent measurement data and to calculate measurement sums and differences. (MP5)

Inquiry Questions:

1. Why is it helpful to organize data in line plots?
2. When might you see fractions in real-world data?
3. Why is it important to establish the whole when plotting fractions on a line plot?
4. How do labels help the reader determine the size of the numbers represented in a line plot?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 3, students represent and interpret data using picture graphs and scaled bar graphs.
3. In Grade 4, this expectation connects with building fractions from unit fractions.
4. In Grade 5, students represent and interpret data with line plots and solve problems using fractional measurements and all four operations.

COLORADO
Department of Education

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

Grade Level Expectation:

4.MD.C. Measurement & Data: Geometric measurement: Understand concepts of angle and measure angles.

Evidence Outcomes

Students Can:

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: (CCSS: 4.MD.C.5)
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. (CCSS: 4.MD.C.5.a)
   b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. (CCSS: 4.MD.C.5.b)
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.C.6)
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (CCSS: 4.MD.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze and measure the size of angles in real-world and mathematical problems. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason abstractly and quantitatively about angles and angular measurement. (MP2)

Inquiry Questions:

1. How is measuring angles with a protractor similar to measuring line segments with a ruler?
2. We can describe the fraction $\frac{3}{100}$ as $\frac{1}{100}+\frac{1}{100}+\frac{1}{100}$. How does this apply to the measurement of angles, such as an angle of 3 degrees?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 4, this expectation connects with drawing and identifying lines and angles, classifying shapes by properties of their lines and angles, and with understanding a fraction as a sum of unit fractions.
3. In Grade 7, students solve real-world and mathematical problems involving angle measure, area, surface area, and volume.

COLORADO Department of Education

Prepared Graduates:

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

Grade Level Expectation:

4.G.A. Geometry: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Evidence Outcomes

Students Can:

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (CCSS: 4.G.A.1)
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS: 4.G.A.2)
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. (CCSS: 4.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make observations and draw conclusions about the classification of two-dimensional figures based on the presence or absence of specified attributes. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use appropriate tools strategically to draw lines (parallel, perpendicular, lines of symmetry), line segments, rays, and angles (right, acute, obtuse). (MP5)
3. Identify ways in which a shape is structured such that it displays line symmetry. (MP7)

Inquiry Questions:

1. Where do you see parallel lines, perpendicular lines, or lines of symmetry in the real world?
2. What kind of angle can you find most often in the real world: right, acute, or obtuse? Why do you think that is the case?
3. What kinds of shapes have many lines of symmetry and what kinds of shapes have no lines of symmetry?
4. In what ways might the lines of symmetry for a shape be related to dividing the shape into fractional parts?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students create composite shapes, recognize and draw shapes having specified attributes, and understand that shapes with shared attributes can define a larger category.
3. In Grade 4, this expectation connects with understanding concepts of angle and measuring angles.
4. In Grade 5, students classify two-dimensional figures into categories based on their properties.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make use of structure.

Grade Level Expectation:

5.NBT.A. Number & Operations in Base Ten: Understand the place value system.

Evidence Outcomes

Students Can:

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. (CCSS: 5.NBT.A.1)
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. (CCSS: 5.NBT.A.2)
3. Read, write, and compare decimals to thousandths. (CCSS: 5.NBT.A.3)
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \frac{1}{10} + 9 \times \frac{1}{100} + 2 \times \frac{1}{1000}$. (CCSS: 5.NBT.A.3.a)
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. (CCSS: 5.NBT.A.3.b)
4. Use place value understanding to round decimals to any place. (CCSS: 5.NBT.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Persist in making sense of how fractions can represent decimal place values. (Personal Skills: Perseverance/Resilience)
2. Abstract place value reasoning with whole numbers to decimal numbers. (MP2)
3. See the structure of place value as not just a making of tens with greater place values, but a making of tenths with lesser place values. (MP7)

Inquiry Questions:

1. How can you show visually the relationships between 25, 2.5 and 0.25? How can you show these relationships with equations?
2. Can all decimals be written as fractions? Why or why not?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 4, students generalize place value understanding for multi-digit whole numbers, use decimal notation for fractions, and compare decimal fractions.
3. In Grade 5, this expectation connects with performing operations with multi-digit whole numbers and operations with decimals to hundredths.
4. In Grade 6, students apply and extend previous understandings of arithmetic to algebraic expressions and develop fluency with decimal operations.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

5.NBT.B. Number & Operations in Base Ten: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Evidence Outcomes

Students Can:

5. Fluently multiply multi-digit whole numbers using the standard algorithm. (CCSS: 5.NBT.B.5)
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 5.NBT.B.6)
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS: 5.NBT.B.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Defend calculations with explanations based on properties of operations, equations, drawings, arrays, and other models. (MP3)
2. Use models and drawings to represent and compute with whole numbers and decimals, illustrating an understanding of place value. (MP5)
3. Use the structure of place value to organize computation with whole numbers and decimals. (MP7)

Inquiry Questions:

1. We sometimes use arrays and area models to model multiplication and division of whole numbers. Do these models work for decimal fractions, too? Why or why not?
2. How is computation with decimal fractions similar to and different from computation with whole numbers?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic.
3. This expectation connects with other ideas in Grade 5: (a) understanding the place value system for decimals, (b) using equivalent fractions as a strategy, (c) applying and extending previous understandings of multiplication and division, and (d) converting like measurement units within a given measurement system.
4. In Grade 6, students compute fluently with multi-digit numbers and find common factors and multiples.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

5.NF.A. Number & Operations—Fractions: Use equivalent fractions as a strategy to add and subtract fractions.

Evidence Outcomes

Students Can:

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. *(In general,* $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$.*)* (CCSS: 5.NF.A.1)
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$, *by observing that* $\frac{3}{7}<\frac{1}{2}$. (CCSS: 5.NF.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Construct viable arguments about the addition and subtraction of fractions with reasoning rooted in the need for like-sized parts. (MP3)
2. Assess the reasonableness of fraction calculations by estimating results using benchmark fractions and number sense. (MP6)
3. Look for structure in the multiplicative relationship between unlike denominators when creating equivalent fractions. (MP7)

Inquiry Questions:

1. It is useful to round decimals when estimating sums and differences of decimal numbers. What would "rounding fractions" look like when estimating sums and differences of fractions?
2. Why don't we add or subtract the denominators when we are working with fractions?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 4, students add and subtract fractions and mixed numbers with like denominators, recognize and generate equivalent fractions, and compare fractions with different numerators and denominators.
3. In Grade 5, this expectation connects with multi-digit whole number operations, operations with decimals to hundredths, and representing and interpreting data.
4. In Grade 6, students reason about and solve one-variable equations and inequalities, and in Grade 7, apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Prepared Graduates:

MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

5.NF.B. Number & Operations—Fractions: Apply and extend previous understandings of multiplication and division.

Evidence Outcomes

Students Can:

3. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (CCSS: 5.NF.B.3)
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (CCSS: 5.NF.B.4)
   a. Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.)* (CCSS: 5.NF.B.4.a)
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS: 5.NF.B.4.b)
5. Interpret multiplication as scaling (resizing), by: (CCSS: 5.NF.B.5)
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. (CCSS: 5.NF.B.5.a)
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1. (CCSS: 5.NF.B.5.b)
6. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.B.6)
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) (CCSS: 5.NF.B.7)
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $\frac{1}{3} \div 4$, and use a visual fraction model to show the quotient. Use the*

relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ *because* $\frac{1}{12} \times 4 = \frac{1}{3}$. (CCSS: 5.NF.B.7.a)

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div \frac{1}{5}$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div \frac{1}{5} = 20$ *because* $20 \times \frac{1}{5} = 4$. (CCSS: 5.NF.B.7.b)

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if* 3 *people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$*-cup servings are in* 2 *cups of raisins?* (CCSS: 5.NF.B.7.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems requiring calculations that scale whole numbers and fractions. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Use fraction models and arrays to interpret and explain fraction calculations. (MP5)
3. Attend carefully to the underlying unit quantities when solving problems involving multiplication and division of fractions. (MP6)
4. Contrast previous understandings of multiplication modeled as equal groups to multiplication as scaling, which is necessary to understand multiplying a fraction or whole number by a fraction, and how the operation of multiplication does not always result in a product larger than both factors. (MP7)

Inquiry Questions:

1. How can you rewrite the fraction $\frac{5}{3}$ with an addition equation? How can you rewrite it with a multiplication equation? How does it make sense that both equations are accurate?
2. If we can describe the product of 5×3 as “three times as big as 5,” what does that tell us about the product of $5 \times \frac{1}{2}$? What about $\frac{1}{5} \times \frac{1}{2}$?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students base understanding of multiplication on its connection to addition, groups of equivalent objects, and area models. In Grade 4, students add and subtract fractions and mixed numbers with like denominators, recognize and generate equivalent fractions, and compare fractions with different numerators and denominators.
3. This expectation connects with several others in Grade 5: (a) performing operations with multi-digit whole numbers and with decimals to hundredths, (b) writing and interpreting numerical expressions, and (c) representing and interpreting data.
4. In Grade 6, students (a) understand ratio concepts and use ratio reasoning to solve problems, (b) apply and extend previous understandings of multiplication and division to divide fractions by fractions, (c) reason about and solve one-variable equations and inequalities, and (d) solve real-world and mathematical problems involving area, surface area, and volume.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

5.OA.A. Operations & Algebraic Thinking: Write and interpret numerical expressions.

Evidence Outcomes

Students Can:

1. Use grouping symbols (parentheses, brackets, or braces) in numerical expressions, and evaluate expressions with these symbols. (CCSS: 5.OA.A.1)
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add* 8 *and* 7*, then multiply by* 2*" as* $2 \times (8 + 7)$. *Recognize that* $3 \times (18932 + 921)$ *is three times as large as* $18932 + 921$*, without having to calculate the indicated sum or product.* (CCSS: 5.OA.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Write expressions that represent mathematical relationships between quantities. (Entrepreneurial Skills: Literacy/Writing)
2. Look for structures and notation that make the order of operations clear when reading and writing mathematical expressions. (MP7)

Inquiry Questions:

1. How can you describe the relationship between the value of $5 \times (24562 + 951)$ and $24562 + 951$ without making any calculations?
2. Suppose we use the letter a to represent a number. Can you determine the relationship between $4 \times a$ and a without knowing the specific number a represents?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 5, this expectation connects with applying and extending previous understandings of multiplication and division.
3. In Grade 6, students compute fluently with multi-digit numbers, find common factors and multiples, and apply and extend previous understandings of arithmetic to algebraic expressions. In middle and high school, students develop fluency with algebraic expressions as mathematical objects that can be used in more complex mathematical operations.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

5.OA.B. Operations & Algebraic Thinking: Analyze patterns and relationships.

Evidence Outcomes

Students Can:

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* (CCSS: 5.OA.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze and compare patterns. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason quantitatively with patterns by relating sequences of numbers with the rule that generated them. (MP3)
3. Look for repeated reasoning both within individual patterns and in mathematical relationships between pairs of patterns. (MP8)

Inquiry Questions:

1. When you graph the corresponding terms formed by two numerical rules, how are the rules reflected in the graph?
2. How does the relationship between two patterns generated by rules relate to the rules themselves?

Coherence Connections:

1. This expectation is in addition to major work of the grade.
2. In Grade 4, students generate and analyze number or shape patterns and generalize about them.
3. In Grade 6, students understand ratio concepts, use ratio reasoning to solve problems, extend previous understandings of arithmetic to algebraic expressions, and represent and analyze quantitative relationships between dependent and independent variables.

Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

5.MD.A. Measurement & Data: Convert like measurement units within a given measurement system.

Evidence Outcomes

Students Can:

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems. (CCSS: 5.MD.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Convert measurements to solve real-world problems. (Professional Skills: Information Literacy)
2. Use appropriate precision when converting measurements based on a problem's context. (MP6)

Inquiry Questions:

1. What is happening mathematically when we convert from centimeters to meters? What about when we convert from meters to centimeters?
2. How can you use fractions to change 53 kilograms to grams? How can you use decimals to do this conversion?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
3. In Grade 5, this expectation connects with performing operations with multi-digit whole numbers and with decimals to hundredths.

Prepared Graduates:

MP5. Use appropriate tools strategically.

Grade Level Expectation:

5.MD.B. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

2. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* (CCSS: 5.MD.B.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Display fractional measurement data in line plots. (Professional Skills: Information Literacy)
2. Participate in discussions of measurement data using information presented in line plots. (Civic/Interpersonal Skills: Literacy/Oral Expression and Listening)
3. Strategically determine the scale of line plots to represent fractional measurements. (MP5)

Inquiry Questions:

1. (Given a data set of fractional measurements with unlike denominators) What will you consider in deciding how to label the tick marks on the line for your line plot?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 4, students represent and interpret data.
3. In Grade 5, this expectation connects with using equivalent fractions and applying and extending previous understandings of multiplication and division.
4. In Grade 6, students develop understanding of statistical variability and summarize and describe distributions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

5.MD.C. Measurement & Data: Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

Evidence Outcomes

Students Can:

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. (CCSS: 5.MD.C.3)
   a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume and can be used to measure volume. (CCSS: 5.MD.C.3.a)
   b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. (CCSS: 5.MD.C.3.b)
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (CCSS: 5.MD.C.4)
5. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. (CCSS: 5.MD.C.5)
   a. Model the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (CCSS: 5.MD.C.5.a)
   b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. (CCSS: 5.MD.C.5.b)
   c. Use the additive nature of volume to find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems. (CCSS: 5.MD.C.5.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve real-world problems involving volume. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make connections between the values being multiplied in a volume formula, the concept of cubic units, and the context within which volume is being calculated. (MP2)
3. Use unit cubes as a tool for finding or estimating volume and compare those results with those obtained with formulas. (MP5)
4. Extend the structure of two-dimensional space and the relationship between arrays and area to three-dimensional space and the relationship between layers of cubes and volume. (MP7)

Inquiry Questions:

1. How are volume and area related in a solid figure?
2. Why is multiplication used when computing the volume of a solid figure, instead of another operation?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students connect area to the operation of multiplication and understand how to represent area problems as multiplication equations.
3. In Grade 6, students solve real-world and mathematical problems involving area of right rectangular prisms with fractional side lengths, using fractional cubic units.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP5. Use appropriate tools strategically.

Grade Level Expectation:

5.G.A. Geometry: Graph points on the coordinate plane to solve real-world and mathematical problems.

Evidence Outcomes

Students Can:

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). (CCSS: 5.G.A.1)
2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (CCSS: 5.G.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use the first quadrant of the coordinate plane to represent real-world and mathematical problems. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Analyze and use information presented visually in a coordinate plane. (Entrepreneurial Skills: Literacy/Reading)
3. Reason quantitatively about a problem by abstracting and representing the situation in the first quadrant of the coordinate plane. (MP2)
4. Use the first quadrant of the coordinate plane as a tool to represent, analyze, and solve problems. (MP5)

Inquiry Questions:

1. What are things in the real world that are designed like a coordinate plane or that use a coordinate system?
2. Why are the axes of the coordinate plane made to form right angles instead of acute and obtuse angles?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students use number lines to represent whole and fractional number distances from zero.
3. In Grade 6, students extend the number line and the coordinate plane to include negative numbers and solve real-world and mathematical problems by graphing points in all four quadrants.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

Grade Level Expectation:

5.G.B. Geometry: Classify two-dimensional figures into categories based on their properties.

Evidence Outcomes

Students Can:

3. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* (CCSS: 5.G.B.3)
4. Classify two-dimensional figures in a hierarchy based on properties. (CCSS: 5.G.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Observe and analyze attributes of two-dimensional figures to classify them. (Entrepreneurial Skills: Inquiry/Analysis)
2. Critique the reasoning of others' classifications of two-dimensional shapes. (MP3)
3. Strategically use measurement tools to help improve the classification of shapes. (MP5)
4. Look for and use attributes of two-dimensional shapes to classify the shapes in a hierarchy of figures. (MP7)

Inquiry Questions:

1. How can you use the words "always," "sometimes," and "never" to develop a classification of two-dimensional figures?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 4, students draw and identify lines and angles and classify shapes by properties of their lines and angles.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

6.RP.A. Ratios & Proportional Relationships: Understand ratio concepts and use ratio reasoning to solve problems.

Evidence Outcomes

Students Can:

1. Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was* $2:1$*, because for every* 2 *wings there was* 1 *beak." "For every vote Candidate* A *received, Candidate* C *received nearly three votes."* (CCSS: 6.RP.A.1)
2. Apply the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of* 3 *cups of flour to* 4 *cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid $75 for* 15 *hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)* (CCSS: 6.RP.A.2)
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (CCSS: 6.RP.A.3)
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (CCSS: 6.RP.A.3.a)
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took* 7 *hours to mow* 4 *lawns, then at that rate, how many lawns could be mowed in* 35 *hours? At what rate were lawns being mowed?* (CCSS: 6.RP.A.3.b)
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent. (CCSS: 6.RP.A.3.c)
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (CCSS: 6.RP.A.3.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use ratio tables to test solutions and determine equivalent ratios. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Analyze and use appropriate quantities and pay attention to units in problems that require reasoning with ratios. (MP2)
3. Construct arguments that compare quantities using ratios or rates. (MP3)
4. Use tables, tape diagrams, and double number line diagrams to provide a structure for seeing equivalency between ratios. (MP7)

Inquiry Questions:

1. How are ratios different from fractions?
2. What is the difference between a quantity and a number?
3. How is a percent also a ratio?
4. How is a rate similar to and also different from a unit rate?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In prior grades, students work with multiplication, division, and measurement. Prior knowledge with the structure of the multiplication table is an important connection for students in creating and verifying equivalent ratios written in symbolic form or in ratio tables (multiplicative comparison vs. additive comparison).
3. In Grade 6, this expectation connects with one-variable equations, inequalities, and representing and analyzing quantitative relationships between dependent and independent variables.
4. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems. In high school, students generalize rates of change to linear and nonlinear functions and use them to describe real-world scenarios.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

6.NS.A. The Number System: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Evidence Outcomes

Students Can:

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for* $\frac{2}{3} \div \frac{3}{4}$ *and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that* $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ *because* $\frac{3}{4}$ *of* $\frac{8}{9}$ *is* $\frac{2}{3}$. *(In general,* $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$.*) How much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{3}{4}$*-cup servings are in* $\frac{2}{3}$ *of a cup of yogurt? How wide is a rectangular strip of land with length* $\frac{3}{4}$ *mi and area* $\frac{1}{2}$ *square mi?* (CCSS: 6.NS.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Create and solve word problems using division of fractions, understanding the relationship of the arithmetic to the problem being solved. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Reason about the contextualized meaning of numbers in word problems involving division of fractions, and decontextualize those numbers to perform efficient calculations. (MP2)
3. Model real-world situations involving scaling by non-whole numbers using multiplication and division by fractions. (MP4)

Inquiry Questions:

1. When dividing, is the quotient always going to be a smaller number than the dividend? Why or why not?
2. What kinds of real-world situations require the division of fractions?
3. How can the division of fractions be modeled visually?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students apply and extend previous understandings of multiplication and division to divide whole numbers by unit fractions and unit fractions by whole numbers.
3. In Grade 6, this expectation connects with solving one-step, one-variable equations and inequalities.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Prepared Graduates:

MP6. Attend to precision.

MP7. Look for and make use of structure.

Grade Level Expectation:

6.NS.B. The Number System: Compute fluently with multi-digit numbers and find common factors and multiples.

Evidence Outcomes

Students Can:

2. Fluently divide multi-digit numbers using the standard algorithm. (CCSS: 6.NS.B.2)
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (CCSS: 6.NS.B.3)
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers $1–100$ with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* $36 + 8$ *as* $4(9 + 2)$. (CCSS: 6.NS.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Accurately add, subtract, multiply, and divide with decimals. (MP6)
2. Recognize the structures of factors and multiples when identifying the greatest common factor and least common multiple of two whole numbers. Use the greatest common factor to rewrite an expression using the distributive property. (MP7)

Inquiry Questions:

1. How do operations with decimals compare and contrast to operations with whole numbers?
2. How does rewriting the sum of two whole numbers using the distributive property yield new understanding and insights on the sum?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 5, students divide whole numbers with two-digit divisors and perform operations with decimals.
3. In Grade 6, this expectation connects with applying and extending previous understandings of arithmetic to algebraic expressions.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. Students apply the concept of greatest common factor to factor linear expressions, and extending properties of whole numbers to variable expressions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

Grade Level Expectation:

6.NS.C. The Number System: Apply and extend previous understandings of numbers to the system of rational numbers.

Evidence Outcomes

Students Can:

5. Explain why positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (CCSS: 6.NS.C.5)
6. Describe a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. (CCSS: 6.NS.C.6)
   a. Use opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; identify that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. (CCSS: 6.NS.C.6.a)
   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; explain that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. (CCSS: 6.NS.C.6.b)
   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. (CCSS: 6.NS.C.6.c)
7. Order and find absolute value of rational numbers. (CCSS: 6.NS.C.7)
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* -3 *is located to the right of* -7 *on a number line oriented from left to right.* (CCSS: 6.NS.C.7.a)
   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3^{\circ}\mathrm{C} > -7^{\circ}\mathrm{C}$ *to express the fact* $-3^{\circ}\mathrm{C}$ *is warmer than* $-7^{\circ}\mathrm{C}$*.* (CCSS: 6.NS.C.7.b)
   c. Define the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* -30 *dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.* (CCSS: 6.NS.C.7.c)
   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than* -30 *dollars represents a debt greater than* 30 *dollars.* (CCSS: 6.NS.C.7.d)
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS: 6.NS.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate integers to form hypotheses, make observations and draw conclusions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Understand the relationship among negative numbers, positive numbers, and absolute value. (MP2)
3. Explain the order of rational numbers using their location on the number line. (MP3)
4. Demonstrate how to plot points on a number line and plot ordered pairs on a coordinate plane. (MP5)

Inquiry Questions:

1. Why do we have negative numbers?
2. What relationships exist among positive and negative numbers on the number line?
3. How does the opposite of a number differ from the absolute value of that same number?
4. How does an ordered pair correspond to its given point on a coordinate plane?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students develop understanding of fractions as numbers and graph points on the coordinate plane (limited to the first quadrant) to solve real-world and mathematical problems.
3. In Grade 6, this expectation connects with reasoning about and solving one-step, one-variable equations and inequalities.
4. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. In Grade 8, students investigate patterns of association in bivariate data.

COLORADO Department of Education

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

6.EE.A. Expressions & Equations: Apply and extend previous understandings of arithmetic to algebraic expressions.

Evidence Outcomes

Students Can:

1. Write and evaluate numerical expressions involving whole-number exponents. (CCSS: 6.EE.A.1)
2. Write, read, and evaluate expressions in which letters stand for numbers. (CCSS: 6.EE.A.2)
   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract* y *from* 5*" as* $5 - y$. (CCSS: 6.EE.A.2.a)
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.* (CCSS: 6.EE.A.2.b)
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = \frac{1}{2}$. (CCSS: 6.EE.A.2.c)
3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression* $3y$. (CCSS: 6.EE.A.3)
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* $y + y + y$ *and* $3y$ *are equivalent because they name the same number regardless of which number* y *stands for.* (CCSS: 6.EE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that expressions can be written in multiple forms and describe cause-and-effect relationships and patterns. (Entrepreneurial Skills: Critical Thinking/Problem Solving and Inquiry/Analysis)
2. Communicate a justification of why expressions are equivalent using arguments about properties of operations and whole numbers. (MP3)
3. See the structure of an expression like $x + 2$ as a sum but also as a single factor in the product $3(x + 2)$. (MP7)
4. Recognize equivalence in variable expressions with repeated addition (such as $y + y + y = 3y$) and repeated multiplication (such as $y \times y \times y = y^3$) and use arithmetic operations to justify the equivalence. (MP8)

Inquiry Questions:

1. How are algebraic expressions similar to and different from numerical expressions?
2. What does it mean for two variable expressions to be equivalent?
3. How might the application of the order of operations differ when using grouping symbols, such as parentheses, for numerical expressions as compared to algebraic expressions?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students understand and apply properties of operations, relationships between inverse arithmetic operations, and write and interpret numerical expressions.
3. In Grade 6, this expectation connects to fluency with multi-digit numbers, finding common factors and multiples, and one-variable equations and inequalities.
4. In future grades, students work with radicals and integer exponents and interpret the structure of more complex algebraic expressions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP6. Attend to precision.

Grade Level Expectation:

6.EE.B. Expressions & Equations: Reason about and solve one-variable equations and inequalities.

Evidence Outcomes

Students Can:

5. Describe solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS: 6.EE.B.5)
6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (CCSS: 6.EE.B.6)
7. Solve real-world and mathematical problems by writing and solving equations of the form $x \pm p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (CCSS: 6.EE.B.7)
8. Write an inequality of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ to represent a constraint or condition in a real-world or mathematical problem. Show that inequalities of the form $x > c$, $x \geq c$, $x < c$, or $x \leq c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS: 6.EE.B.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate unknown values to form hypotheses, make observations, and draw conclusions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason about the values and operations of an equation both within a real-world context and abstracted from it. (MP2)
3. State precisely the meaning of variables used when setting up equations. (MP6)

Inquiry Questions:

1. What are the different ways a variable can be used in an algebraic equation or inequality? For example, how are these uses of the variable x different from each other? (a) $x + 5 = 8$; (b) $x = \frac{1}{2}$; (c) $x > 5$.
2. How is the solution to an inequality different than a solution to an equation?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students write simple expressions that record calculations with numbers, interpret numerical expressions without evaluating them, and generate ordered pairs from two numerical rules.
3. This expectation connects to several others in Grade 6: (a) understanding ratio concepts and use ratio reasoning to solve problems, (b) applying and extending previous understandings of multiplication and division to divide fractions by fractions, (c) applying and extending previous understandings of numbers to the system of rational numbers, (d) applying and extending previous understandings of arithmetic to algebraic expressions, and (e) representing and analyzing quantitative relationships between dependent and independent variables.
4. In Grade 7, students solve real-life and mathematical problems involving two-step equations and inequalities. In Grade 8, students work with radicals and integer exponents and solve linear equations and pairs of simultaneous linear equations.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

6.EE.C. Expressions & Equations: Represent and analyze quantitative relationships between dependent and independent variables.

Evidence Outcomes

Students Can:

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.* (CCSS: 6.EE.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze relationships between dependent and independent variables. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason about the operations that relate constant and variable quantities in equations with dependent and independent variables. (MP2)
3. Model with mathematics by describing real-world situations with equations and inequalities. (MP4)

Inquiry Questions:

1. How can you determine if a variable is the independent variable or the dependent variable?
2. What are the advantages of showing the relationship between an independent and dependent variable in multiple representations (table, graph, equation)?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students analyze numerical patterns and relationships, including generating and graphing ordered pairs in the first quadrant.
3. In Grade 6, this expectation connects with understanding ratio concepts and using ratio reasoning to solve problems.
4. In Grade 7, students decide if two quantities are in a proportional relationship and identify the unit rate in tables, graphs, equations, diagrams, and verbal descriptions.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

6.SP.A. Statistics & Probability: Develop understanding of statistical variability.

Evidence Outcomes

Students Can:

1. Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* (CCSS: 6.SP.A.1)
2. Demonstrate that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape. (CCSS: 6.SP.A.2)
3. Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (CCSS: 6.SP.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Identify statistical questions that require the collection of data representing multiple perspectives. (Civic/Interpersonal Skills: Global/Cultural Awareness)
2. Make sense of practical problems by turning them into statistical investigations. (MP1)
3. Reason abstractly and quantitatively with data collected from statistical investigations by describing the data's center, spread, and shape. (MP2)
4. Model variability in data collected to answer statistical questions and draw conclusions based on center, spread, and shape. (MP4)

Inquiry Questions:

1. What distinguishes a statistical question from a question that is not a statistical question?
2. Why do we have numerical measures for both the center of a data set and the variability of a data set?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students represent and interpret data in dot plots/line plots, and use arithmetic to answer questions about the plots.
3. In Grade 6, students summarize and describe data distributions using numerical measures of center and spread, and terms such as cluster, peak, gap, symmetry, skew, and outlier.
4. In Grade 7, students use random sampling to draw inferences about a population and draw informal comparative inferences about two populations. In high school, students summarize, represent, and interpret data on a single count or measurement variable.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

6.SP.B. Statistics & Probability: Summarize and describe distributions.

Evidence Outcomes

Students Can:

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (CCSS: 6.SP.B.4)
5. Summarize numerical data sets in relation to their context, such as by: (CCSS: 6.SP.B.5)
   a. Reporting the number of observations. (CCSS: 6.SP.B.5.a)
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. (CCSS: 6.SP.B.5.b)
   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (CCSS: 6.SP.B.5.c)
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS: 6.SP.B.5.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Write informative texts describing statistical distributions, their measures, and how they relate to the context in which the data were gathered. (Entrepreneurial Skills: Literacy/Writing)
2. Move from context to abstraction and back to context while finding and using measures of center and variability and describing what they mean in the context of the data. (MP2)
3. Analyze data sets and draw conclusions based on the data display and measures of center and/or variability. (MP4)
4. Identify clusters, peaks, gaps, and symmetry, and describe the meaning of these and other patterns in data distributions. (MP7)

Inquiry Questions:

1. How can different data displays communicate different meanings?
2. When is it better to use the mean as a measure of center? Why?
3. When is it better to use the median as a measure of center? Why?
4. How many values of a data set do you use to calculate the range? Interquartile range? Mean absolute deviation? How does this help to compare what these measures represent?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students represent and interpret data in dot plots and line plots.
3. In Grade 6, this expectation connects with developing understanding of statistical variability.
4. In high school, students summarize, represent, and interpret data on a single count or measurement variable (including standard deviation), and make inferences and justify conclusions from sample surveys, experiments, and observational studies.

COLORADO Department of Education

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

6.G.A. Geometry: Solve real-world and mathematical problems involving area, surface area, and volume.

Evidence Outcomes

Students Can:

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.1)
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.2)
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.3)
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS: 6.G.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that problems can be identified and possible solutions can be created with respect to using area, surface area, and volume. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of a problem by understanding the context of the problem before applying a formula. (MP1)
3. Model real-world problems involving shape and space. (MP4)
4. Strategically use coordinate planes, nets of three-dimensional figures, and area and volume formulas as tools to solve real-world problems. (MP5)

Inquiry Questions:

1. What is the difference between what area measures and what volume measures?
2. How does using decomposition aid in finding the area of composite figures?
3. How are nets of three-dimensional figures used to find surface area?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 4, students solve problems involving measurement and converting from a larger unit to a smaller unit. In Grade 5, students understand concepts of volume, relate volume to multiplication and to addition, and graph points on the coordinate plane to solve real-world and mathematical problems.
3. In Grade 6, this expectation connects with graphing points in all four quadrants of the coordinate plane and finding distances between points with the same first coordinate or the same second coordinate.
4. In Grade 7, students draw, construct, and describe geometrical figures and describe the relationships between them, and solve real-world and mathematical problems involving angle measure, area, surface area, and volume. In Grade 8, students understand congruence and similarity and understand and apply the Pythagorean Theorem.

COLORADO Department of Education

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

7.RP.A. Ratios & Proportional Relationships: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Evidence Outcomes

Students Can:

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.* (CCSS: 7.RP.A.1)
2. Identify and represent proportional relationships between quantities. (CCSS: 7.RP.A.2)
   a. Determine whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (CCSS: 7.RP.A.2.a)
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. (CCSS: 7.RP.A.2.b)
   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* (CCSS: 7.RP.A.2.c)
   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where r is the unit rate. (CCSS: 7.RP.A.2.d)
3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (CCSS: 7.RP.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize when proportional relationships occur and apply these relationships to personal experiences. (Entrepreneurial Skills: Inquiry/Analysis)
2. Recognize, identify, and solve problems that involve proportional relationships to make predictions and describe associations among variables. (MP1)
3. Reason quantitatively with rates and their units in proportional relationships. (MP2)
4. Use repeated reasoning to test for equivalent ratios, such as reasoning that walking $\frac{1}{2}$ mile in $\frac{1}{4}$ hour is equivalent to walking 1 mile in $\frac{1}{2}$ hour and equivalent to walking 2 miles in 1 hour, the unit rate. (MP8)

Inquiry Questions:

1. How are proportional relationships related to unit rates?
2. How can proportional relationships be expressed using tables, equations, and graphs?
3. What are properties of all proportional relationships when graphed on the coordinate plane?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students understand ratio concepts and use ratio reasoning to solve problems.
3. This expectation connects with several others in Grade 7: (a) solving real-life and mathematical problems using numerical and algebraic expressions and equations, (b) investigating chance processes and developing, using, and evaluating probability models, and (c) drawing, constructing, and describing geometrical figures and describing the relationships between them.
4. In Grade 8, students (a) understand the connections between proportional relationships, lines, and linear equations, (b) define, evaluate, and compare functions, and (c) use functions to model relationships between quantities. In high school, students use proportional relationships to define trigonometric ratios, solve problems involving right triangles, and find arc lengths and areas of sectors of circles.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP7. Look for and make use of structure.

Grade Level Expectation:

7.NS.A. The Number System: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Evidence Outcomes

Students Can:

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. (CCSS: 7.NS.A.1)
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* (CCSS: 7.NS.A.1.a)
   b. Demonstrate $p+q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.1.b)
   c. Demonstrate subtraction of rational numbers as adding the additive inverse, $p-q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. (CCSS: 7.NS.A.1.c)
   d. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS: 7.NS.A.1.d)
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. (CCSS: 7.NS.A.2)
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1)=1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.2.a)
   b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left(\frac{p}{q}\right)=\frac{-p}{q}=\frac{p}{-q}$. Interpret quotients of rational numbers by describing real-world contexts. (CCSS: 7.NS.A.2.b)
   c. Apply properties of operations as strategies to multiply and divide rational numbers. (CCSS: 7.NS.A.2.c)
   d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. (CCSS: 7.NS.A.2.d)
3. Solve real-world and mathematical problems involving the four operations with rational numbers. *(Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)* (CCSS: 7.NS.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems with rational numbers using all four operations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Compute with rational numbers abstractly and interpret quantities in context. (MP2)
3. Justify understanding and computational accuracy of operations with rational numbers. (MP3)
4. Use additive inverses, absolute value, the distributive property, and properties of operations to reason with and operate on rational numbers. (MP7)

Inquiry Questions:

1. How do operations with integers compare to and contrast with operations with whole numbers?
2. How can operations with negative integers be modeled visually?
3. How can it be determined if the decimal form of a rational number terminates or repeats?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students use the four operations with whole numbers and fractions to solve problems.
3. In Grade 7, this expectation connects with solving real-life and mathematical problems using numerical and algebraic expressions and equations. This expectation begins the formal study of rational numbers (a number expressible in the form $\frac{a}{b}$ or $-\frac{a}{b}$ for some fraction $\frac{a}{b}$; the rational numbers include the integers) as extended from their study of fractions, which in these standards always refers to non-negative numbers.
4. In Grade 8, students extend their study of the real number system to include irrational numbers, radical expressions, and integer exponents. In high school, students work with rational exponents and complex numbers.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

7.EE.A. Expressions & Equations: Use properties of operations to generate equivalent expressions.

Evidence Outcomes

Students Can:

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. (CCSS: 7.EE.A.1)
2. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by* 5%*" is the same as "multiply by* 1.05*."* (CCSS: 7.EE.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize that the structures of equivalent algebraic expressions provide different ways of seeing the same problem. (MP7)

Inquiry Questions:

1. How is it determined that two algebraic expressions are equivalent?
2. What is the value of having an algebraic expression in equivalent forms?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students apply and extend previous understandings of arithmetic to algebraic expressions.
3. In Grade 8, students use equivalent expressions to analyze and solve linear equations and pairs of simultaneous linear equations. In high school, students use equivalent expressions within various families of functions to reveal key features of graphs and how those features are related to contextual situations.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.

Grade Level Expectation:

7.EE.B. Expressions & Equations: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Evidence Outcomes

Students Can:

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making* $\$25$ *an hour gets a* 10% *raise, she will make an additional* $\frac{1}{10}$ *of her salary an hour, or* $\$2.50$*, for a new salary of* $\$27.50$*. If you want to place a towel bar* $9\frac{3}{4}$ *inches long in the center of a door that is* $27\frac{1}{2}$ *inches wide, you will need to place the bar about* 9 *inches from each edge; this estimate can be used as a check on the exact computation.* (CCSS: 7.EE.B.3)
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. (CCSS: 7.EE.B.4)
   a. Solve word problems leading to equations of the form $px \pm q = r$ and $p(x \pm q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is* 54 *cm. Its length is* 6 *cm. What is its width?* (CCSS: 7.EE.B.4.a)
   b. Solve word problems leading to inequalities of the form $px \pm q > r$, $px \pm q \geq r$, $px \pm q < r$, or $px \pm q \leq r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid* $\$50$ *per week plus* $\$3$ *per sale. This week you want your pay to be at least* $\$100$*. Write an inequality for the number of sales you need to make and describe the solutions.* (CCSS: 7.EE.B.4.b)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Adapt to different forms of equations and inequalities and reach solutions that make sense in context. (Personal Skills: Adaptability/Flexibility)
2. Use mental computation and estimation to check the reasonableness of their solutions. Make connections between the sequence of operations used in an algebraic approach and an arithmetic approach, understanding how simply reasoning about the numbers connects to writing and solving a corresponding algebraic equation. (MP1)
3. Represent a situation symbolically and solve, attending to the meaning of quantities and variables. (MP2)
4. Select an appropriate solution approach (calculator, mental math, drawing a diagram, etc.) based on the specific values and/or desired result of a problem. (MP5)
5. Use estimation, mental calculations, and understanding of real-world contexts to assess the reasonableness of answers to real-life and mathematical problems. (MP6)

Inquiry Questions:

1. Do the properties of operations apply to variables the same way they do to numbers? Why?
2. Why are there different ways to solve equations?
3. In what scenarios might estimation be better than an exact answer?
4. How can the reasonableness of a solution be determined?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 6, students reason about and solve one-step, one-variable equations and inequalities.
3. In Grade 7, this expectation connects with analyzing proportional relationships, using them to solve real-world and mathematical problems, and applying and extending previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
4. In Grade 8, students work with radicals and integer exponents, analyze and solve linear equations and pairs of simultaneous linear equations, and describe functional relationships.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

Grade Level Expectation:

7.SP.A. Statistics & Probability: Use random sampling to draw inferences about a population.

Evidence Outcomes

Students Can:

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; explain that generalizations about a population from a sample are valid only if the sample is representative of that population. Explain that random sampling tends to produce representative samples and support valid inferences. (CCSS: 7.SP.A.1)
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* (CCSS: 7.SP.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Infer about a population using a random sample. (Entrepreneurial Skills: Inquiry/Analysis)
2. Make conjectures about population parameters and support arguments with sample data. (MP3)
3. Use multiple samples to informally model the variability of sample statistics like the mean. (MP4)

Inquiry Questions:

1. Why would a researcher use sampling for a study or survey?
2. Why does random sampling give more trustworthy results than nonrandom sampling in a study or survey? How might methods for obtaining a sample for a study or survey affect the results of the survey?
3. How can a winner be concluded in an election, from a sample, before counting all the ballots?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 6, students develop understanding of statistical variability.
3. In Grade 7, this expectation connects with drawing informal comparative inferences about two populations, investigating chance processes, and with developing, using, and evaluating probability models.
4. In high school, students understand and evaluate random processes underlying statistical experiments and also make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.

Grade Level Expectation:

7.SP.B. Statistics & Probability: Draw informal comparative inferences about two populations.

Evidence Outcomes

Students Can:

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* (CCSS: 7.SP.B.3)
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* (CCSS: 7.SP.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Interpret variability in statistical distributions and draw conclusions about the distance between their centers using units of mean absolute deviation. (Entrepreneurial Skills: Inquiry/Analysis)
2. Base arguments about the difference between two distributions on the relative variability of the distributions, not just the difference between the two distribution means. (MP3)
3. Model real-world populations with statistical distributions and compare the distributions using measures of center and variability. (MP4)

Inquiry Questions:

1. How do measures of center (such as mean) and variability (such as mean absolute deviation) work together to describe comparisons of data?
2. How can we use measures of center and variability to compare two data sets? Why is it not wise to compare two data sets using only measures of center?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 6, students study measures of center and variability to describe, compare, and contrast data sets.
3. In Grade 7, this expectation connects with using random sampling to draw inferences about a population.
4. In high school, students summarize, represent, and interpret data on a single count or measurement variable and also make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Prepared Graduates:

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

Grade Level Expectation:

7.SP.C. Statistics & Probability: Investigate chance processes and develop, use, and evaluate probability models.

Evidence Outcomes

Students Can:

5. Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS: 7.SP.C.5)
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube* 600 *times, predict that a* 3 *or* 6 *would be rolled roughly* 200 *times, but probably not exactly* 200 *times.* (CCSS: 7.SP.C.6)
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. (CCSS: 7.SP.C.7)
   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* (CCSS: 7.SP.C.7.a)
   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* (CCSS: 7.SP.C.7.b)
8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (CCSS: 7.SP.C.8)
   a. Explain that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. (CCSS: 7.SP.C.8.a)
   b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. (CCSS: 7.SP.C.8.b)
   c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If* 40% *of donors have type A blood, what is the probability that it will take at least* 4 *donors to find one with type A blood?* (CCSS: 7.SP.C.8.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Be innovative when designing simulations to generate frequencies of compound events by using random digits, dice, coins, or other chance objects to represent the probabilities of real-world events. (Entrepreneurial Skills: Creativity/Innovation)
2. Use probability models and simulations to predict outcomes of real-world chance events both theoretically and experimentally. (MP4)
3. Use technology, manipulatives, and simulations to determine probabilities and understand chance events. (MP5)

Inquiry Questions:

1. Since the probability of getting heads on the toss of a fair coin is $\frac{1}{2}$, does that mean for every one hundred tosses of a coin exactly fifty of them will be heads? Why or why not?
2. What might a discrepancy in the predicted outcome and the actual outcome of a chance event tell us?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In prior grades, students study rational numbers and operations with rational numbers.
3. In Grade 7, probability concepts support the major work of understanding rational numbers. This expectation connects with analyzing proportional relationships, using them to solve real-world and mathematical problems, and using random sampling to draw inferences about a population.
4. In high school, students understand and evaluate random processes underlying statistical experiments, understand independence and conditional probability and use them to interpret data, and use the rules of probability to compute probabilities of compound events in a uniform probability model.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

Grade Level Expectation:

7.G.A. Geometry: Draw, construct, and describe geometrical figures and describe the relationships between them.

Evidence Outcomes

Students Can:

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS: 7.G.A.1)
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS: 7.G.A.2)
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in cross sections of right rectangular prisms and right rectangular pyramids. (CCSS: 7.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate what side and angle measurements are necessary to determine a unique triangle. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason abstractly by deconstructing three-dimensional shapes into two-dimensional cross-sections. (MP2)
3. Describe, analyze, and generalize about the resulting cross-section of a sliced three-dimensional figure and justify their reasoning. (MP3)
4. Appropriately use paper, pencil, ruler, compass, protractor, or technology to draw geometric shapes. (MP5)

Inquiry Questions:

1. How are proportions used to solve problems involving scale drawings?
2. What are some examples of cross-sections whose shapes may be identical but are from different three-dimensional figures?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 6, students solve real-world and mathematical problems involving area, surface area, and volume.
3. In Grade 7, this expectation connects with analyzing proportional relationships and using them to solve real-world and mathematical problems.
4. In Grade 8, students understand the connections between proportional relationships, lines, and linear equations, and understand congruence and similarity using physical models, transparencies, or geometry software. In high school, students use geometric constructions as a basis for geometric proof.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP6. Attend to precision.

Grade Level Expectation:

7.G.B. Geometry: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Evidence Outcomes

Students Can:

4. State the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (CCSS: 7.G.B.4)
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure. (CCSS: 7.G.B.5)
6. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS: 7.G.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving angle measure, area, surface area, and volume. (Entrepreneurial Skills: Inquiry/Analysis)
2. Persevere with complex shapes by analyzing their component parts and applying geometric properties and measures of area and volume. (MP1)
3. Model real-world situations involving area, surface area, and volume. (MP4)
4. Reason accurately with measurement units when calculating angles, circumference, area, surface area, and volume. (MP6)

Inquiry Questions:

1. How can the formula for the area of a circle be derived from the formula for the circumference of the circle?
2. What are the angle measure relationships in supplementary, complementary, vertical, and adjacent angles?
3. What are some examples of real-world situations where one would need to find (a) area, (b) volume, and (c) surface area?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students understand concepts of angle, measure angles, and solve real-world and mathematical problems involving area, surface area, and volume.
3. In Grade 8, students understand congruence and similarity using physical models, transparencies, or geometry software, and understand and apply the Pythagorean Theorem. Students also use the formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems.

Prepared Graduates:

MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.NS.A. The Number System: Know that there are numbers that are not rational, and approximate them by rational numbers.

Evidence Outcomes

Students Can:

1. Demonstrate informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Define irrational numbers as numbers that are not rational. (CCSS: 8.NS.A.1)
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS: 8.NS.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate rational and irrational numbers and their relative approximate positions on a number line. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use technology to look for repetition in decimal expansions and use number lines to order and compare irrational numbers relative to rational numbers. (MP5)
3. Apply understanding of rational and irrational numbers to describe and work within the structure of the real number system effectively and efficiently. (MP7)
4. Recognize repetition in decimal expansions of rational numbers and recognize when a decimal expansion cannot be represented by a rational number. (MP8)

Inquiry Questions:

1. How many irrational numbers exist?
2. Why is there no real number closest to zero?
3. Can you accurately plot an irrational number on the number line? How do you know?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
3. In Grade 8, this expectation supports working with radicals and integer exponents. This concludes the introduction of all numbers that comprise the real number system.
4. In high school, students use properties of rational and irrational numbers, work with rational exponents, and extend their understanding of number systems to include complex numbers.

Prepared Graduates:

MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.EE.A. Expressions & Equations: Work with radicals and integer exponents.

Evidence Outcomes

Students Can:

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (CCSS: 8.EE.A.1)
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares (up to 100) and cube roots of small perfect cubes (up to 64). Know that $\sqrt{2}$ is irrational. (CCSS: 8.EE.A.2)
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3 *times* 10^8 *and the population of the world as* 7 *times* 10^9*, and determine that the world population is more than* 20 *times larger.* (CCSS: 8.EE.A.3)
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (CCSS: 8.EE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason about unusually large or small quantities. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use calculators or computers to compute with and approximate radicals and roots, and understand how such tools represent scientific notation. (MP5)
3. Explore the structure of numerical expressions with integer exponents to generate equivalent expressions. (MP7)
4. Look for how positive integer exponents are equivalent to repeated multiplication by the base and how negative integer exponents are equivalent to repeated division by the base. (MP8)

Inquiry Questions:

1. How is performing operations on numbers in scientific notation similar to or different from performing operations on numbers in standard notation?
2. Why does a positive number raised to a negative exponent not equal a negative number?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students use whole-number exponents to denote powers of ten. In Grades 6 and 7, they work with algebraic and numerical expressions containing whole-number exponents.
3. In Grade 8, this expectation connects with knowing that there are numbers that are not rational and approximating them by rational numbers, understanding and applying the Pythagorean Theorem, and solving real-world and mathematical problems involving volume of cylinders, cones, and spheres. In high school, students extend work with exponents to rational exponents.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.EE.B. Expressions & Equations: Understand the connections between proportional relationships, lines, and linear equations.

Evidence Outcomes

Students Can:

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (CCSS: 8.EE.B.5)
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (CCSS: 8.EE.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make connections between representations of linear growth. (Entrepreneurial Skills: Inquiry/Analysis)
2. Record information about constant rates of change in graphs and equations. (Entrepreneurial Skills: Literacy/Writing)
3. Make sense of and compare proportional relationships represented in different forms. (MP1)
4. Compare, contrast, and make claims with proportional relationships based on properties of equations, tables, and/or graphs. (MP3)
5. Explore the structure of proportional relationships expressed as equations or graphs for methods of comparison. (MP7)

Inquiry Questions:

1. How is the unit rate of a proportional relationship related to the slope of its graphical representation?
2. Why are similar triangles effective for describing slope geometrically?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students recognize and represent proportional relationships, calculate the constant of proportionality, and graph proportional relationships on the coordinate plane, recognizing that they always pass through the origin.
3. In Grade 8, this expectation connects with analyzing and solving linear equations and pairs of simultaneous linear equations, with defining, evaluating, and comparing functions, and with understanding congruence and similarity using physical models, transparencies, or geometry software.
4. In high school, students compare multiple representations of inverse proportional relationships.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.EE.C. Expressions & Equations: Analyze and solve linear equations and pairs of simultaneous linear equations.

Evidence Outcomes

Students Can:

7. Solve linear equations in one variable. (CCSS: 8.EE.C.7)
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (CCSS: 8.EE.C.7.a)
    b. Solve linear equations with rational number coefficients, including equations with variables on both sides and whose solutions require expanding expressions using the distributive property and collecting like terms. (CCSS: 8.EE.C.7.b)
8. Analyze and solve pairs of simultaneous linear equations. (CCSS: 8.EE.C.8)
    a. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (CCSS: 8.EE.C.8.a)
    b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be* 5 *and* 6. (CCSS: 8.EE.C.8.b)
    c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (CCSS: 8.EE.C.8.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving linear equations and systems of linear equations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Solve problems that require a system of linear equations in two variables. (MP1)
3. Model real-world problems with linear equations and systems of linear equations, with variables defined in their real-world context. (MP4)
4. Solve equations and systems of equations and express solutions with accuracy that makes sense in the real-world context modeled by the equations. (MP6)
5. Recognize the structure of equations and of systems of equations that produce one, infinitely many, or no solution. (MP7)

Inquiry Questions:

1. What is meant by a "solution" to a linear equation? What is meant by a "solution" to a system of two linear equations? How are these concepts related?
2. How is it possible for an equation to have more than one solution? How is it possible for an equation to have no solution?
3. Why can't a system of linear equations have a solution set other than one, zero, or infinitely many solutions?
4. What connections exist between the graphical solution and the algebraic solution of a system of linear equations?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students reason about and solve one-step and two-step, one-variable equations and inequalities, use properties of operations to generate equivalent expressions, and solve real-world and mathematical problems using numerical and algebraic expressions and equations.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations and with investigating patterns of association in bivariate data.
4. In high school, students abstract and generalize about linear functions and how they compare and contrast to nonlinear functions. Students also reason about and solve systems of equations that include one or more nonlinear equations.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.F.A. Functions: Define, evaluate, and compare functions.

Evidence Outcomes

Students Can:

1. Define a function as a rule that assigns to each input exactly one output. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required for Grade 8.) (CCSS: 8.F.A.1)
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (CCSS: 8.F.A.2)
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points* $(1,1)$, $(2,4)$ *and* $(3,9)$, *which are not on a straight line.* (CCSS: 8.F.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make connections between the information gathered through tables, equations, graphs, and verbal descriptions of functions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Define variables as quantities and interpret ordered pairs from a functional relationship with respect to those variables. (MP2)
3. With and without technology, analyze and describe functions that are not linear with the use of equations, graphs, and tables. (MP5)
4. See a function as a rule that assigns each input to exactly one output; this structure does not "turn inputs into outputs"; rather, it describes the relationship between items in two sets. (MP7)
5. Recognize patterns of linear growth in different representations of linear functions. (MP8)

Inquiry Questions:

1. Why is it important to know if a mathematical relationship is a function?
2. How can you determine if a function is linear or nonlinear?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations and with using functions to model relationships between quantities.
4. In high school, students use function notation, analyze functions using different representations, build new functions from existing functions, and extend from linear functions to quadratic, exponential, and other more advanced functions.

Prepared Graduates:

MP4. Model with mathematics.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.F.B. Functions: Use functions to model relationships between quantities.

Evidence Outcomes

Students Can:

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS: 8.F.B.4)
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS: 8.F.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Describe in writing the qualitative features of linear or nonlinear functions. (Entrepreneurial Skills: Literacy/Writing)
2. Model real-world situations with linear functions. (MP4)
3. Explore properties of linear functions and how those properties appear in the structure of linear equations in slope-intercept form. (MP7)
4. Use strategies to calculate the rate of change in a linear function (slope) and use properties of linear functions to create equations. (MP8)

Inquiry Questions:

1. What is the minimum information needed to write a linear function for a relationship between two quantities?
2. What are some quantitative and qualitative features of graphs of functions?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems.
3. In Grade 8, this expectation connects with defining, evaluating, and comparing functions and with investigating patterns of association in bivariate data.
4. In high school, students use function notation, analyze functions using different representations, build new functions from existing functions, and extend from linear functions to quadratic, exponential, and other more advanced functions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.SP.A. Statistics & Probability: Investigate patterns of association in bivariate data.

Evidence Outcomes

Students Can:

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (CCSS: 8.SP.A.1)
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (CCSS: 8.SP.A.2)
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of* 1.5 *cm/hr as meaning that an additional hour of sunlight each day is associated with an additional* 1.5 *cm in mature plant height.* (CCSS: 8.SP.A.3)
4. Explain that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* (CCSS: 8.SP.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize and describe patterns in bivariate data. (Entrepreneurial Skills: Inquiry/Analysis)
2. Interpret the contextual meaning of slope and y-intercept, where applicable in a linear model fit to bivariate data. (MP2)
3. Build statistical models to explore, describe, and generalize the relationship between two variables. (MP4)
4. Use scatter plots and two-way tables to describe possible associations in bivariate data. (MP7)

Inquiry Questions:

1. In what ways is a scatter plot useful in describing and interpreting the relationship between two quantities?
2. Why would we create a linear model for a set of bivariate data?
3. How do you know when a credible prediction can be made from a linear model of bivariate data?
4. What does a pattern of association look like for categorical data?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In previous grades, students apply and extend previous understandings of numbers to the system of rational numbers.
3. In Grade 8, this expectation supports using functions to model relationships between quantities.
4. In high school, students summarize, represent, and interpret data on two categorical and quantitative variables, interpret linear models, and understand independence and conditional probability.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.G.A. Geometry: Understand congruence and similarity using physical models, transparencies, or geometry software.

Evidence Outcomes

Students Can:

1. Verify experimentally the properties of rotations, reflections, and translations: (CCSS: 8.G.A.1)
   a. Lines are taken to lines, and line segments to line segments of the same length. (CCSS: 8.G.A.1.a)
   b. Angles are taken to angles of the same measure. (CCSS: 8.G.A.1.b)
   c. Parallel lines are taken to parallel lines. (CCSS: 8.G.A.1.c)
2. Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (CCSS: 8.G.A.2)
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (CCSS: 8.G.A.3)
4. Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (CCSS: 8.G.A.4)
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* (CCSS: 8.G.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Think about how rotations, reflections, and translations of a geometric figure preserve congruence as similar to how properties of operations such as the associative, commutative, and distributive properties preserve equivalence of arithmetic and algebraic expressions. (Entrepreneurial Skills: Critical Thinking/Problem Solving and Inquiry/Analysis)
2. Explain a sequence of transformations that results in a congruent or similar triangle. (MP3)
3. Use physical models, transparencies, geometric software, or other appropriate tools to explore the relationships between transformations and congruence and similarity. (MP5)
4. Use the structure of the coordinate system to describe the locations of figures obtained with rotations, reflections, and translations. (MP7)
5. Reason that since any one rotation, reflection, or translation of a figure preserves congruence, then any sequence of those transformations must also preserve congruence. (MP8)

Inquiry Questions:

1. How are properties of rotations, reflections, translations, and dilations connected to congruence?
2. How are properties of rotations, reflections, translations, and dilations connected to similarity?
3. Why are angle measures significant regarding the similarity of two figures?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students solve problems involving angle measure, area, surface area, and volume, and draw, construct, and also describe geometrical figures and the relationships between them.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations.
4. In high school, students extend their work with transformations, apply the concepts of transformations to prove geometric theorems, and use similarity to define trigonometric functions.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.G.B. Geometry: Understand and apply the Pythagorean Theorem.

Evidence Outcomes

Students Can:

6. Explain a proof of the Pythagorean Theorem and its converse. (CCSS: 8.G.B.6)
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS: 8.G.B.7)
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS: 8.G.B.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Think of the Pythagorean Theorem as not just a formula, but a formula that only holds true under certain conditions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Construct a viable argument about why a proof of the Pythagorean Theorem is valid. (MP3)
3. Test to see if a triangle is a right triangle by applying the Pythagorean Theorem. (MP7)
4. Use patterns to recognize and generate Pythagorean triples. (MP8)

Inquiry Questions:

1. What is the relationship between the Pythagorean Theorem and its converse? In what ways is each useful?
2. Is it always possible to use the Pythagorean Theorem to find the distance between points on the coordinate plane? How do you know?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grades 6 and 7, students solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
3. In Grade 8, this expectation connects with radicals and integer exponents, square roots, and solving simple equations in the form $x^2 = p$.
4. In high school, students (a) prove and apply trigonometric identities, (b) prove theorems involving similarity, (c) define trigonometric ratios and solve problems involving right triangles, (d) translate between the geometric description and the equation for a conic section, and (e) use coordinates to prove simple geometric theorems algebraically.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

Grade Level Expectation:

8.G.C. Geometry: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Evidence Outcomes

Students Can:

9. State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (CCSS: 8.G.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Efficiently solve problems using established volume formulas. (Professional Skills: Task/Time Management)
2. Describe how the formulas for volumes of cones, cylinders, and spheres relate to one another and to the volume formulas for solids with rectangular bases. (MP3)
3. Use appropriate precision when solving problems involving measurements and volume formulas that describe real-world shapes. (MP6)

Inquiry Questions:

1. How are the formulas of cones, cylinders, and spheres similar to each other?
2. How are the formulas of cones, cylinder, and spheres connected to the formulas for pyramids, prisms, and cubes?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 7, students solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
3. In Grade 8, this expectation connects with radicals and integer exponents.
4. In high school, students apply geometric concepts in mathematical modeling situations and to solve design problems.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.N-RN.A. The Real Number System: Extend the properties of exponents to rational exponents.

Evidence Outcomes

Students Can:

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{\frac{1}{3}}$ *to be the cube root of* 5 *because we want* $\left(5^{\frac{1}{3}}\right)^3 = 5^{\left(\frac{1}{3}\right)3}$ *to hold, so* $\left(5^{\frac{1}{3}}\right)^3$ *must equal* 5. (CCSS: HS.N-RN.A.1)
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. (CCSS: HS.N-RN.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly reason with rational exponents by applying properties of exponents to create equivalent expressions. (Personal Skills: Adaptability/Flexibility)
2. Make explicit connections between integer and rational exponents, and express how radical notation connects to rational exponents. (MP3)
3. Generalize the properties of integer exponents to rational exponents, and apply these properties to a wider variety of situations, such as rewriting the formula for the volume of a sphere of radius r, $V = \frac{4}{3}\pi r^3$ to express the radius in terms of the volume, $r = \left(\frac{3}{4} \times \frac{V}{\pi}\right)^{\frac{1}{3}}$. (MP7)

Inquiry Questions:

1. How do we know that the two equations below both represent the same relationship, between the radius of a sphere and its volume? $V = \frac{4}{3}\pi r^3$ and $r = \left(\frac{3}{4} \times \frac{V}{\pi}\right)^{\frac{1}{3}}$
2. How does the property of exponents $(x^a)^b = x^{ab}$ help us make sense of the meaning of rational exponents?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grade 8, students study integer exponents (both positive and negative) and radicals. Here, students expand the concept of exponents to include rational exponents and make the connection to radicals.
3. Much of high school mathematics is based on the assumption that the properties of rational numbers extend to irrational numbers, and this understanding allows students to view the full picture of the real number system. Students' understanding of the number line is a reasonable way to understand that the behavior of irrational numbers isn't different than that of rational numbers, as both densely populate the number line.
4. In advanced courses, rational exponents extends to irrational exponents by means of exponential and logarithmic functions.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.N-RN.B. The Real Number System: Use properties of rational and irrational numbers.

Evidence Outcomes

Students Can:

3. (+) Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. (CCSS: HS.N-RN.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Express understanding of rational and irrational numbers verbally and in writing in ways appropriate to the context and audience. (Civic/Interpersonal Skills: Communication)
2. Justify conclusions about rational and irrational numbers and communicate them to others. (MP3)
3. Accurately use vocabulary describing rational and irrational numbers. (MP6)
4. Generalize the properties of integer exponents to rational exponents. (MP7)

Inquiry Questions:

1. How is it possible that multiplying two irrational numbers gives a product that is not irrational? Why doesn't this phenomenon apply to rational numbers?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. Having already extended arithmetic from whole numbers to fractions (Grades 4–6) and from fractions to rational numbers (Grade 7), students in Grade 8 encountered particular irrational numbers such as $\sqrt{5}$. In high school, students use and understand the real number system in a variety of contexts.
3. An important difference between rational and irrational numbers is that rational numbers form a number system. If you add, subtract, multiply, or divide two rational numbers, you get another rational number (provided the divisor is not 0 in the last case). The same is not true of irrational numbers. Although in applications of mathematics the distinction between rational and irrational numbers is irrelevant, since we always deal with finite decimal approximations (and therefore with rational numbers), thinking about the properties of rational and irrational numbers is good practice for mathematical reasoning habits such as constructing viable arguments (MP3) and attending to precision (MP6).

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP6. Attend to precision.

Grade Level Expectation:

HS.N-Q.A. Quantities: Reason quantitatively and use units to solve problems.

Evidence Outcomes

Students Can:

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (CCSS: HS.N-Q.A.1)
2. Define appropriate quantities for the purpose of descriptive modeling. (CCSS: HS.N-Q.A.2)
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (CCSS: HS.N-Q.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason with units to solve problems. (Professional Skills: Information Literacy)
2. Create a coherent representation of problems by considering the units involved, attending to the meaning of quantities and not just how to compute them, and flexibly using properties of operations and objects. (MP2)
3. Use care in specifying units and in selecting units to label axes. (MP6)

Inquiry Questions:

1. In what ways can the units of a complicated problem help guide us to the solution?
2. How can units be used to explain your work or to critique the work of others?
3. Why is “Let x = number of gallons of gas” more accurate than “Let x = gas?”

Coherence Connections:

1. This expectation represents major work of high school.
2. In middle school, students worked with measurement units, including units obtained by multiplying and dividing quantities. In high school, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight.
3. A central theme in applied mathematics and everyday life is units. For example, acceleration, currency conversions, energy, power, density, and social science rates (e.g., number of deaths per 100,000) all require an understanding of units.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.N-CN.A. The Complex Number System: Perform arithmetic operations with complex numbers.

Evidence Outcomes

Students Can:

1. Define complex number i such that $i^2 = -1$, and show that every complex number has the form $a + bi$ where a and b are real numbers. (CCSS: HS.N-CN.A.1)
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. (CCSS: HS.N-CN.A.2)
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. (CCSS: HS.N-CN.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems that previously appeared to have no solutions by performing operations with complex numbers. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Extend an understanding of the number system to include imaginary and complex numbers and recognize the underlying structures that connect them to the real number system. (MP7)

Inquiry Questions:

1. Is the sum of two complex numbers always, sometimes, or never a complex number? Why or why not?
2. Is the product of two complex numbers always, sometimes, or never a complex number? Why or why not?

Coherence Connections:

1. This expectation is in addition to the major work of high school and includes an advanced (+) outcome.
2. In Grade 8, students evaluate square roots and use them to represent the solution of an equation in the form $x^2 = p$ where p is a positive rational number. The complex numbers extend that knowledge to introduce the solution of equations in the form $x^2 = p$ where p is negative rational number.
3. During the years from kindergarten to Grade 8, students must repeatedly extend their conception of number. With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Prepared Graduates:

MP6. Attend to precision.

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.N-CN.B. The Complex Number System: Represent complex numbers and their operations on the complex plane.

Evidence Outcomes

Students Can:

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. (CCSS: HS.N-CN.B.4)
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $\left(-1+\sqrt{3i}\right)^3 = 8$ *because* $\left(-1+\sqrt{3i}\right)$ *has modulus* 2 *and argument* $120°$. (CCSS: HS.N-CN.B.5)
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. (CCSS: HS.N-CN.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use the complex plane in rectangular and polar form to represent complex numbers. (Professional Skills: Information Literacy)
2. Accurately add, subtract, multiply, conjugate, and calculate distance using complex numbers. (MP6)
3. Represent complex numbers in rectangular and polar form. (MP7)

Inquiry Questions:

1. How can the rectangular and polar forms of real and imaginary numbers represent the same number?
2. How is distance on the complex plane similar to and different from distance on the Cartesian coordinate system?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. Beginning in Grade 5, students plot points on the coordinate plane. By Grade 8, students graph the solutions to linear equations and linear inequalities. The complex plane extends that knowledge to include complex solutions.
3. Beyond high school, complex numbers are used in advanced work such as the study of quantum physics and modeling AC electricity.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.N-CN.C. The Complex Number System: Use complex numbers in polynomial identities and equations.

Evidence Outcomes

Students Can:

7. Solve quadratic equations with real coefficients that have complex solutions. (CCSS: HS.N-CN.C.7)
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite as* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. (CCSS: HS.N-CN.C.8)
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (CCSS: HS.N-CN.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of equations that have real and complex solutions. (MP1)
2. Use spreadsheets, graphing tools, and other technology to understand quadratic equations with real coefficients and complex solutions. (MP5)
3. Recognize the properties of a quadratic equation that indicate complex solutions without having to compute the solutions. (MP7)

Inquiry Questions:

1. What differences are evident in the graph of a quadratic equation with real solutions versus the graph of a quadratic equation with complex solutions?

Coherence Connections:

1. This expectation supports the major work of high school and includes advanced (+) outcomes.
2. In high school, students extend the real numbers to complex numbers, and one effect is that they now have a complete theory of quadratic equations: Every quadratic equation with complex coefficients has (counting multiplicities) two roots in the complex numbers, and students are able to express any quadratic polynomial as the product of linear factors.
3. With an understanding of the complex number system, students can now make sense of the Fundamental Theorem of Algebra, which states that every non-constant single-variable polynomial with complex coefficients has at least one complex root.

Prepared Graduates:

MP4. Model with mathematics.

MP6. Attend to precision.

Grade Level Expectation:

HS.N-VM.A. Vector & Matrix Quantities: Represent and model with vector quantities.

Evidence Outcomes

Students Can:

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $|\mathbf{v}|$, $\|\mathbf{v}\|$, v). (CCSS: HS.N-VM.A.1)
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. (CCSS: HS.N-VM.A.2)
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors. (CCSS: HS.N-VM.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use vector quantities to represent both magnitude and direction. (Professional Skills: Information Literacy)
2. Model real-world forces and other quantities with vectors. (MP4)
3. Accurately represent magnitude and direction when graphing and describing vectors. (MP6)

Inquiry Questions:

1. How do vectors represent magnitude and direction?
2. What information does a velocity vector provide?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. In advanced mathematics courses, students apply their understanding of vectors to physics and engineering.

Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

HS.N-VM.B. Vector & Matrix Quantities: Perform operations on vectors.

Evidence Outcomes

Students Can:

4. (+) Add and subtract vectors. (CCSS: HS.N-VM.B.4)
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. (CCSS: HS.N-VM.B.4.a)
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. (CCSS: HS.N-VM.B.4.b)
   c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. (CCSS: HS.N-VM.B.4.c)
5. (+) Multiply a vector by a scalar. (CCSS: HS.N-VM.B.5)
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g. as $\mathrm{c}(\mathrm{v}_x, \mathrm{v}_y) = (\mathrm{cv}_x, \mathrm{cv}_y)$. (CCSS: HS.N-VM.B.5.a)
   b. Compute the magnitude of a scalar multiple $\mathrm{c}\mathbf{v}$ using $\|\mathrm{c}\mathbf{v}\| = |\mathrm{c}|\mathbf{v}$. Compute the direction of $\mathrm{c}\mathbf{v}$ knowing that when $|\mathrm{c}|\mathbf{v} \neq 0$, the direction of $\mathrm{c}\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$). (CCSS: HS.N-VM.B.5.b)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve real-world problems using operations on vectors. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Accurately represent magnitude and direction when using vector arithmetic. (MP6)

Inquiry Questions:

1. Why isn't vector addition simply a matter of adding the magnitudes of the vectors?
2. How does a scalar change the direction and magnitude of a vector?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. In advanced mathematics courses, students may apply vectors to problems in engineering, physics, and meteorology.

Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

HS.N-VM.C. Vector & Matrix Quantities: Perform operations on matrices and use matrices in applications.

Evidence Outcomes

Students Can:

6. (+) Use matrices to represent and manipulate data, e.g., as when all of the payoffs or incidence relationships in a network. (CCSS: HS.N-VM.C.6)
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. (CCSS: HS.N-VM.C.7)
8. (+) Add, subtract, and multiply matrices of appropriate dimensions. (CCSS: HS.N-VM.C.8)
9. (+) Understand that, unlike the multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. (CCSS: HS.N-VM.C.9)
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. (CCSS: HS.N-VM.C.10)
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimension to produce another vector. Work with matrices as transformations of vectors. (CCSS: HS.N-VM.C.11)
12. (+) Work with 2×2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. (CCSS: HS.N-VM.C.12)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Understand how matrices can represent systems of equations and are useful when systems contain too many variables to efficiently operate on without technology. (Professional Skills: Information Literacy)
2. Accurately represent addition, subtraction, multiplication, and the identity matrix when using matrix arithmetic. (MP6)

Inquiry Questions:

1. What information can be modeled using matrices?
2. What is the role of dimension in matrix operations?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. In advanced mathematics courses, students may apply matrices to model circuits in electronics.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.A-SSE.A. Seeing Structure in Expressions: Interpret the structure of expressions.

Evidence Outcomes

Students Can:

1. Interpret expressions that represent a quantity in terms of its context.★ (CCSS: HS.A-SSE.A.1)
   a. Interpret parts of an expression, such as terms, factors, and coefficients. (CCSS: HS.A-SSE.A.1.a)
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* (CCSS: HS.A-SSE.A.1.b)
2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (CCSS: HS.A-SSE.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Interpret expressions and their parts. (Entrepreneurial Skills: Inquiry/Analysis)
2. Make sense of variables, constants, constraints, and relationships in the context of a problem. (MP1)
3. Think abstractly about how terms in an expression can be rewritten or how terms can be combined and treated as a single object to be computed with. (MP2)
4. Discern a pattern or structure to see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. (MP7)

Inquiry Questions:

1. How could you show algebraically that the two expressions $(n+2)^2 - 4$ and $n^2 + 4n$ are equivalent? How could you show it visually, with a diagram or picture?

Coherence Connections:

1. This expectation represents major work of high school and includes a modeling (★) outcome.
2. In Grades 6 and 7, students use the properties of operations to generate equivalent expressions.
3. In high school, students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in intentional manipulation of algebraic expressions, and strategically using different representations.
4. The separation of algebra and functions in the Standards is intended to specify the difference between the two, as mathematical concepts between expressions and equations on the one hand and functions on the other. Students often enter college-level mathematics courses conflating all three of these.

Prepared Graduates:

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

HS.A-SSE.B. Seeing Structure in Expressions: Write expressions in equivalent forms to solve problems.

Evidence Outcomes

Students Can:

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ (CCSS: HS.A-SSE.B.3)
   a. Factor a quadratic expression to reveal the zeros of the function it defines. (CCSS: HS.A-SSE.B.3.a)
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. (CCSS: HS.A-SSE.B.3.b)
   c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* $1.15t$ *can be rewritten as* $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* (CCSS: HS.A-SSE.B.3.c)
4. Use the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. *For example, calculate mortgage payments.*★ (CCSS: HS.A-SSE.B.4)
   a. (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1). (CCSS: HS.A-SSE.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Transform expressions to highlight properties and set up solution strategies. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Recognize the difference in the structure of linear, quadratic, and other equations and apply the appropriate strategies to solve. (MP7)
3. Notice, for example, the regularity in the way terms combine to make zero when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$, and how it might lead to the general formula for the sum of a finite geometric series. (MP8)

Inquiry Questions:

1. What does the vertex form of a quadratic equation, $y=a(x-h)^2+k$, tell us about its graph that the standard form, $y=ax^2+bx+c$, does not?
2. What does the factored form of a quadratic equation, $y=a(x-p)(x-q)$, tell us about the graph that the other two forms do not?

Coherence Connections:

1. This expectation represents major work of high school and includes modeling (★) and advanced (+) outcomes.
2. In middle school, students manipulate algebraic expressions to create equivalent expressions. In high school, students' manipulations become more strategic and advanced in response to increasingly complex expressions.
3. As students progress through high school, they should become increasingly proficient with mathematical actions such as "doing and undoing"; for example, looking at expressions generated through the distributive property and identifying expressions that might have led to a given outcome. They do not use "FOIL" as a justification for the multiplication of two binomials, understanding that such mnemonics are not conceptually defensible and do not generalize.

Prepared Graduates:

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

HS.A-APR.A. Arithmetic with Polynomials & Rational Expressions: Perform arithmetic operations on polynomials.

Evidence Outcomes

Students Can:

1. Explain that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. (CCSS: HS.A-APR.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Students make hypotheses and draw conclusions about polynomials and operations on them. (Entrepreneurial Skills: Inquiry/Analysis)
2. Understand how types of numbers and operations form a closed system. (MP7)
3. See how operations on polynomials yield polynomials, much like how operations on integers yield integers. (MP8)

Inquiry Questions:

1. $f(x) = x^2$ is a nonnegative polynomial because for all values of x, $f(x) \geq 0$. If you add two nonnegative polynomials together, do you always, sometimes, or never get another nonnegative polynomial? What if you multiply them?

Coherence Connections:

1. This expectation represents major work of high school.
2. In previous grades, students understand algebraic expressions as values in which one or more letters are used to stand for an unspecified or unknown number and use the properties of operations to write expressions in different but equivalent forms.
3. In high school, polynomials and rational expressions form a system in which they can be added, subtracted, multiplied, and divided. Polynomials are analogous to the integers; rational expressions are analogous to the rational numbers.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.A-APR.B. Arithmetic with Polynomials & Rational Expressions: Understand the relationship between zeros and factors of polynomials.

Evidence Outcomes

Students Can:

2. Know and apply the Remainder Theorem. For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. (Students need not apply the Remainder Theorem to polynomials of degree greater than 4.) (CCSS: HS.A-APR.B.2)
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. (CCSS: HS.A-APR.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Understand that the zeros of a polynomial that models a real-world context generally convey useful information about that context. (Professional Skills: Information Literacy)
2. Make sense of the relationship between zeros and factors of polynomials using graphs, tables, real-world contexts, and equations in factored forms. (MP1)
3. Reason with a factored quadratic, such as $(x + 2)(x - 3) = 0$, abstractly as “something times zero is zero” and “zero times something is zero.” (MP2)
4. Look for the ways the structure of polynomial equations are different from linear equations and use appropriate methods, such as factoring, to reveal the polynomial’s zeros. (MP7)

Inquiry Questions:

1. What is the relationship between factoring a polynomial and finding its zeros?

Coherence Connections:

1. This expectation represents major work of high school.
2. In previous grades, students rewrite algebraic expressions in equivalent forms.
3. In high school, students construct polynomial functions with specified zeros. This is the first step in a progression that can lead, as an extension topic, to constructing polynomial functions whose graphs pass through any specified set of points in the plane.
4. A particularly important application of polynomial division is the case where a polynomial $p(x)$ is divided by a linear factor of the form $(x - a)$ for a real number a. In this case, the remainder is the value $p(a)$ of the polynomial at $x = a$. This topic should not be reduced to “synthetic division,” which reduces the method to a matter of carrying numbers between registers while obscuring the reasoning that makes the result evident. It is important to regard the Remainder Theorem as a theorem, not a technique.
5. Experience with constructing polynomial functions satisfying given conditions is useful preparation not only for calculus (where students learn more about approximating functions), but for understanding the mathematics behind curve-fitting methods used in applications of statistics and computer graphics.

Prepared Graduates:

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

HS.A-APR.C. Arithmetic with Polynomials & Rational Expressions: Use polynomial identities to solve problems.

Evidence Outcomes

Students Can:

4. (+) Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* (CCSS: HS.A-APR.C.4)
5. (+) Know and apply the Binomial Theorem for the expansion of in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.) (CCSS: HS.A-APR.C.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Understand the connections between the coefficients in expansions of $(x + y)^n$ and the values in Pascal's Triangle. (Entrepreneurial Skills: Inquiry/Analysis)
2. Recognize patterns in the binomial coefficients as they appear in Pascal's Triangle. (MP8)

Inquiry Questions:

1. Can you find a case (a specific value of x and y) for which the equation $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ does not generate a Pythagorean triple?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. Polynomials form a rich ground for mathematical explorations that reveal relationships in the system of integers.

COLORADO Department of Education

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.A-APR.D. Arithmetic with Polynomials & Rational Expressions: Rewrite rational expressions.

Evidence Outcomes

Students Can:

6. Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. (CCSS: HS.A-APR.D.6)
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expressions; add, subtract, multiply, and divide rational expressions. (CCSS: HS.A-APR.D.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason with rational expressions like $\frac{x^2+5x+6}{x+2}$ not as a sum divided by a sum, but as a yet-to-be-factored numerator where one of the factors, $(x+2)$, will make 1 when divided by the denominator. (MP2)
2. Determine when it is appropriate to use a computer algebra system or calculator instead of paper and pencil to rewrite rational expressions. (MP5)
3. Understand how types of numbers and operations form a closed system. (MP7)

Inquiry Questions:

1. How is dividing polynomials like and unlike long division with whole numbers?

Coherence Connections:

1. This expectation represents major work of high school and includes an advanced (+) outcome.
2. The analogy between polynomials and integers also applies to polynomial and integer division. Students should recognize the high school method of polynomial long division to find quotients and remainders of polynomials as similar to the method of integer long division first experienced in Grade 4.
3. In high school, polynomials and rational expressions form a system in which they can be added, subtracted, multiplied, and divided. Polynomials are analogous to the integers; rational expressions are analogous to the rational numbers.
4. Expressing rational expressions in different forms allows students to see different properties of the graph, such as horizontal asymptotes.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.A-CED.A. Creating Equations: Create equations that describe numbers or relationships.★

Evidence Outcomes

Students Can:

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* (CCSS: HS.A-CED.A.1)
2. Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. (CCSS: HS.A-CED.A.2)
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* (CCSS: HS.A-CED.A.3)
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R. (CCSS: HS.A-CED.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason contextually, within the real-world context of the problem, and decontextually, about the mathematics, without regard to the context. (MP2)
2. Model and solve problems arising in everyday life, society, and the workplace. Interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (MP4)
3. Use pencil and paper, concrete models, a ruler, a calculator, a spreadsheet, a computer algebra system, and/or dynamic geometry software to make sense of and solve mathematical equations. (MP5)
4. Use the structure of an equation and a sequence of operations to rearrange the equation to isolate a variable by itself on one side of the equal sign. (MP7)

Inquiry Questions:

1. What are some similarities and differences in creating equations of different types?
2. What features of a real-world context might indicate that the equation that models it is quadratic instead of linear?

Coherence Connections:

1. This expectation represents major work of high school and includes modeling (★) outcomes.
2. In previous grades, students model real-world situations with mathematics. Modeling becomes a major objective in high school, including not only an increase in the complexity of the equations studied, but an upgrade of the student's ability in every part of the modeling cycle.
3. The repertoire of functions that is acquired during high school allows students to create more complex equations, including equations arising from linear and quadratic expressions, and simple rational and exponential expressions. Students in high school start using parameters in their equations, to represent whole classes of equations or to represent situations where the equation is to be adjusted to fit data.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.A-REI.A. Reasoning with Equations & Inequalities: Understand solving equations as a process of reasoning and explain the reasoning.

Evidence Outcomes

Students Can:

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (CCSS: HS.A-REI.A.1)
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. (CCSS: HS.A-REI.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Articulate steps of solving an equation using written communication skills. (Civic/Interpersonal Skills: Communication)
2. Describe a logical flow of mathematics, using stated assumptions, definitions, and previously established results in constructing arguments, and explain solving equations as a process of reasoning that demystifies “extraneous” solutions that can arise under certain solution procedures. (MP3)
3. Understand that solving equations is a process of reasoning where properties of operations can be used to change expressions on either side of the equation to equivalent expressions. (MP7)

Inquiry Questions:

1. What types of equations can have extraneous solutions? What types cannot? Why?
2. How are extraneous solutions generated?

Coherence Connections:

1. This expectation represents major work of high school.
2. In previous grades, students solve equations and inequalities.
3. In high school, students extend their skills with solving equations and inequalities to generalize about the solution methods themselves. They name assumptions, justify their steps, and view the process through the lens of proof rather than simple obtaining of a solution.
4. Students’ understanding of solving equations as a process of reasoning demystifies extraneous solutions that can arise under certain solution procedures. The reasoning begins from the assumption that x is a number that satisfies the equation and ends with a list of possibilities for x. But not all the steps are necessarily reversible, and so it is not necessarily true that every number in the list satisfies the equation.

Prepared Graduates:

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.A-REI.B. Reasoning with Equations & Inequalities: Solve equations and inequalities in one variable.

Evidence Outcomes

Students Can:

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (CCSS: HS.A-REI.B.3)
4. Solve quadratic equations in one variable. (CCSS: HS.A-REI.B.4)
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. (CCSS: HS.A-REI.B.4.a)
   b. Solve quadratic equations (e.g., for $x^2 = 49$) by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. (CCSS: HS.A-REI.B.4.b)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve equations and draw conclusions from their solutions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Strategically use calculators or computer technology, and recognize instances when the form of the equation doesn't lend itself to using these tools. (MP5)
3. Analyze the structure of a quadratic equation to determine the most efficient solution strategy. (MP7)

Inquiry Questions:

1. How does the initial form of a quadratic equation cue us to an appropriate solution strategy?

Coherence Connections:

1. This expectation represents major work of high school.
2. With an understanding of solving equations as a reasoning process, students can organize the various methods for solving different types of equations into a coherent picture. For example, solving linear equations involves only steps that are reversible (adding a constant to both sides, multiplying both sides by a non-zero constant, transforming an expression on one side into an equivalent expression). Therefore, solving linear equations does not produce extraneous solutions. The process of completing the square also involves only this same list of steps, and so converts any quadratic equation into an equivalent equation of the form $(x-p)^2 = q$ that has exactly the same solutions.
3. It is traditional for students to spend a lot of time on various techniques of solving quadratic equations, which are often presented as if they are completely unrelated (factoring, completing the square, the quadratic formula). In fact, the key step in completing the square involves factoring and the quadratic formula is nothing more than an encapsulation of the method of completing the square, expressing the actions repeated in solving a collection of quadratic equations with numerical coefficients with a single formula. Rather than long drills on techniques of dubious value, students with an understanding of the underlying reasoning behind all these methods are opportunistic in their application, choosing the method that best suits the situation at hand.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

HS.A-REI.C. Reasoning with Equations & Inequalities: Solve systems of equations.

Evidence Outcomes

Students Can:

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (CCSS: HS.A-REI.C.5)
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (CCSS: HS.A-REI.C.6)
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ *and the circle* $x^2 + y^2 = 3$. (CCSS: HS.A-REI.C.7)
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. (CCSS: HS.A-REI.C.8)
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater). (CCSS: HS.A-REI.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve systems of equations by using graphs and algebraic methods. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Substitute expressions for variables when solving systems of equations, thinking of the expressions as single objects rather than a process that must be computed before substitution. (MP2)
3. Use a matrix to model a system of equations, which may itself be a model of a real-world situation. (MP4)

Inquiry Questions:

1. How is the solution to a system of equations related to the graph of the system? What if the system has no solution? What if the system has infinitely many solutions?
2. Two lines may intersect in zero, one, or infinitely many points. How many intersections may there be between a line and the graph of a quadratic equation?

Coherence Connections:

1. This expectation represents major work of high school and includes advanced (+) outcomes.
2. In previous grades, students solve systems of two linear equations graphically and by using substitution, and understand the concept of a solution of a system of equations.
3. In high school, students solve systems of equations using methods that include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Students may use graphing calculators or other technology to model and find approximate solutions for systems of equations.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.A-REI.D. Reasoning with Equations & Inequalities: Represent and solve equations and inequalities graphically.

Evidence Outcomes

Students Can:

10. Explain that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (CCSS: HS.A-REI.D.10)
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. (CCSS: HS.A-REI.D.11)
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (CCSS: HS.A-REI.D.12)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze and use information presented in equations and visually in graphs. (Entrepreneurial Skills: Literacy/Reading)
2. Make sense of correspondences between equations, verbal descriptions, tables, and graphs. (MP1)
3. Use graphing calculators and/or computer technology to reason about and solve systems of equations and inequalities. (MP5)
4. Specify units of measure, label axes to clarify the correspondence with quantities in a problem, calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. (MP6)
5. Use the characteristics and structures of function families to understand and generalize about solutions to equations and inequalities. (MP7)

Inquiry Questions:

1. How is a solution to a system of inequalities different than the solution to a system of equations?
2. How are the types of functions in the system related to the number of solutions it might have? Can you give an example to explain your thinking?
3. How many different ways can you find to solve $x^2 = (2x - 9)^2$?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grade 8, students begin their study of systems of equations with systems of two linear equations. With a focus on graphical solutions, they build understanding of the concept of a system and its solution(s) or lack thereof. The concept is built upon in high school, extending to algebraic solution strategies as well as considering solutions of systems of non-linear equations and of inequalities.
3. In high school, students use algebraic solution methods that produce precise solutions and understand that these can be represented graphically or numerically. Students may use graphing calculators or other technology to generate tables of values, graph, and solve systems involving a variety of functions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.F-IF.A. Interpreting Functions: Understand the concept of a function and use function notation.

Evidence Outcomes

Students Can:

1. Explain that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. (CCSS: HS.F-IF.A.1)
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (CCSS: HS.F-IF.A.2)
3. Demonstrate that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$. (CCSS: HS.F-IF.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Describe sequences as functions. (MP2)
2. Use accurate terms and symbols when describing functions and using function notation. (MP6)
3. Understand a function as a correspondence where each element of the domain is assigned to exactly one element of the range; this structure does not "turn inputs into outputs"; rather, it describes the relationship between elements in two sets. (MP7)

Inquiry Questions:

1. Besides the notation we use, what makes a function different from an equation?
2. Why is it important to know if an equation is a function?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grade 8, students define, evaluate and compare functions. Although students are expected to give examples of functions that are not linear functions, linear functions are the focus.
3. In high school, students deepen their understanding of the notion of function, expanding their repertoire to include quadratic and exponential functions.
4. In calculus, the concepts of function together with the rate of change are integral to reason about how variables operate together.

Prepared Graduates:

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.F-IF.B. Interpreting Functions: Interpret functions that arise in applications in terms of the context.

Evidence Outcomes

Students Can:

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* (CCSS: HS.F-IF.B.4)
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ (CCSS: HS.F-IF.B.5)
6. Calculate and interpret the average rate of change presented symbolically or as a table, of a function over a specified interval. Estimate the rate of change from a graph.★ (CCSS: HS.F-IF.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use functions and their graphs to model, interpret, and generalize about real-world situations. (MP4)
2. Graph functions and interpret key features of the graphs or use key features to construct a graph; use technology as a tool to visualize and understand how various functions behave in different representations. (MP5)
3. Make structural comparisons between linear, exponential, quadratic and higher order polynomial, rational, radical and trigonometric functions to describe commonalities, consistencies, and differences. (MP7)

Inquiry Questions:

1. In what ways does a real-world context influence the domain of the function that models it?
2. How are slope and rate of change related?

Coherence Connections:

1. This expectation represents major work of high school and includes modeling (★) outcomes.
2. In Grade 8, students understand the connections between proportional relationships, lines, and linear equations, and analyze and solve linear equations and pairs of simultaneous linear equations.
3. The rate of change of a linear function is equal to the slope of the line that is its graph. And because the slope of a line is constant, that is, between any two points it is the same, "the rate of change" has an unambiguous meaning for a linear function. In high school, the concept of slope is generalized to rates of change. Students understand that for linear functions, the rate of change is a constant, and for nonlinear functions, we refer to average rates of change over a given interval.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

Grade Level Expectation:

HS.F-IF.C. Interpreting Functions: Analyze functions using different representations.

Evidence Outcomes

Students Can:

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ (CCSS: HS.F-IF.C.7)
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima. (CCSS: HS.F-IF.C.7.a)
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (CCSS: HS.F-IF.C.7.b)
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. (CCSS: HS.F-IF.C.7.c)
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. (CCSS: HS.F-IF.C.7.d)
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (CCSS: HS.F-IF.C.7.e)
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (CCSS: HS.F-IF.C.8)
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (CCSS: HS.F-IF.C.8.a)
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^{\frac{t}{10}}$, *and classify them as representing exponential growth or decay.* (CCSS: HS.F-IF.C.8.b)
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (CCSS: HS.F-IF.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason abstractly and understand the connections between the symbolic representation, the table of values, and the key features of the graph of a function. (MP2)
2. Use calculators or other graphing software to explore and analyze the graphs of complex and advanced functions. Use the understanding gained to sketch graphs by hand, when appropriate. (MP5)
3. Attend to important terms, definitions, and symbols when graphing, describing, and writing equivalent forms of functions. (MP6)

Inquiry Questions:

1. How can we rewrite a function to illustrate its key features? Give an example of a function written two different ways, one where one or more key features is evident, and another where they are not.
2. Which types of functions share underlying characteristics? How does this help us understand the function families?

Coherence Connections:

1. This expectation represents major work of high school and includes modeling (★) and advanced (+) outcomes.
2. In Grade 8, students understand the connections between proportional relationships, lines, and linear equations, graph linear equations, and analyze and solve linear equations and pairs of simultaneous linear equations.
3. In high school, students are able to recognize, construct, and apply attributes of exponential and quadratic functions, and also use the families of exponential and quadratic functions in a more general sense as a way to model and explain phenomena.
4. Across high school mathematics courses, students have opportunities to compare and contrast functions as they reason about the structure inherent in functions in general and the structure within specific families of functions. Considering functions with the same domains can be a useful classification for comparing and contrasting.

Prepared Graduates:

MP4. Model with mathematics.

Grade Level Expectation:

HS.F-BF.A. Building Functions: Build a function that models a relationship between two quantities.

Evidence Outcomes

Students Can:

1. Write a function that describes a relationship between two quantities.★ (CCSS: HS.F-BF.A.1)
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context. (CCSS: HS.F-BF.A.1.a)
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (CCSS: HS.F-BF.A.1.b)
   c. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.* (CCSS: HS.F-BF.A.1.c)
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ (CCSS: HS.F-BF.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Students apply their understanding of functions to real-world contexts. (MP4)

Inquiry Questions:

1. Why does a function require one output for every input?
2. How can the ideas of cause and effect be developed through the building of functions?

Coherence Connections:

1. This expectation represents major work of high school and includes modeling (★) and advanced (+) outcomes.
2. In previous grades, students understand how a function is defined and use equations to model relationships between two variables in context.
3. In high school, students build from the understanding of input and output to understanding dependence in mathematical relationships. Work with building functions is closely connected to expectations in the algebra domain, and provides opportunity to apply the modeling cycle (see Appendix).

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.F-BF.B. Building Functions: Build new functions from existing functions.

Evidence Outcomes

Students Can:

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k both positive and negative; find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* (CCSS: HS.F-BF.B.3)
4. Find inverse functions. (CCSS: HS.F-BF.B.4)
    a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = \frac{x+1}{x-1}$ *for* $x \neq 1$. (CCSS: HS.F-BF.B.4.a)
    b. (+) Verify by composition that one function is the inverse of another. (CCSS: HS.F-BF.B.4.b)
    c. (+) Read values of an inverse function from a graph or table, given that the function has an inverse. (CCSS: HS.F-BF.B.4.c)
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain. (CCSS: HS.F-BF.B.4.d)
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. (CCSS: HS.F-BF.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use calculators or computer technology to create, describe, and analyze related functions. (Professional Skills: Use Information and Communication Technologies)
2. Create verbal and written explanations of the generalities they find across and between function families. (MP3)
3. Use accurate terms, definitions and mathematical symbols when building, describing, and manipulating functions. (MP6)
4. Extend the patterns of transformations of functions and make connections between function representations. (MP7)

Inquiry Questions:

1. What is meant by a "function family"?
2. Describe cases where the inverse of a function is only a function when the domain is restricted.

Coherence Connections:

1. This expectation is in addition to the major work of high school and includes advanced (+) outcomes.
2. In previous grades, students understand how a function is defined and describe how the slope and y-intercept of a linear function are evident on the graph of a linear equation.
3. Students develop a notion of naturally occurring families of functions that deserve particular attention. This can come from experimenting with the effect on the graph of simple algebraic transformations of the input and output variables. Quadratic and absolute value functions are good contexts for getting a sense of the effects of many of these transformations, but eventually, students need to understand these ideas abstractly and be able to talk about them for any function f.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.F-LE.A. Linear, Quadratic & Exponential Models: Construct and compare linear, quadratic, and exponential models and solve problems.★

Evidence Outcomes

Students Can:

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. (CCSS: HS.F-LE.A.1)
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (CCSS: HS.F-LE.A.1.a)
   b. Identify situations in which one quantity changes at a constant rate per unit interval relative to another. (CCSS: HS.F-LE.A.1.b)
   c. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (CCSS: HS.F-LE.A.1.c)
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (CCSS: HS.F-LE.A.2)
3. Use graphs and tables to describe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (CCSS: HS.F-LE.A.3)
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. (CCSS: HS.F-LE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason about and with situations that can be modeled by functions. In high school, focused study on multiple function types adds complexity to the reasoning required. (MP1)
2. Use linear, exponential, and logarithmic functions and their properties and graphs to model and reason about real-world situations. (MP4)
3. Distinguish between situations that can be modeled with linear functions and with exponential functions using understandings of rates of growth and factors of growth over equal intervals. (MP7)

Inquiry Questions:

1. In what ways are linear and exponential functions similar? In what ways are quadratic and exponential functions similar?
2. How can observing the connections between table, graph, and function notation help you better understand the function?

Coherence Connections:

1. This expectation represents major work of high school and includes modeling (★) outcomes.
2. In Grade 8, students understand that the ratio of the rise and run for any two distinct points on a line is the same and that this concept is referred to as slope or as rate of change.
3. To support the high school major work of deeply understanding functions, students' work with linear and exponential functions as models of real-world phenomena lends itself to reasoning, analysis, comparison, and generalizations about linear and exponential functions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

HS.F-LE.B. Linear, Quadratic, & Exponential Models: Interpret expressions for functions in terms of the situation they model.★

Evidence Outcomes

Students Can:

5. Interpret the parameters in a linear or exponential function in terms of a context. (CCSS: HS.F-LE.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of a mathematical model of a real-world situation and describe and interpret its meaning both mathematically and contextually. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Both decontextualize—abstract a given situation and representing it symbolically and manipulate the representing symbols without necessarily attending to their referents—and contextualize—pause as needed during the manipulation process in order to probe into the referents for the symbols involved. (MP2)
3. Use mathematics to model, interpret, and reason about real-world contexts. (MP4)

Inquiry Questions:

1. What does the linear component, $bx + c$, of a quadratic expression determine about the quadratic function?
2. How do the a and b values in the exponential function $f(x) = ab^x$ compare to the a and b values in the linear function $g(x) = a + bx$?

Coherence Connections:

1. This expectation represents major work of high school and includes a modeling (★) outcome.
2. In Grade 8, students model linear relationships with functions with graphs and tables.
3. In high school, students describe rate of change between two quantities as well as initial values both within and apart from context. An understanding of how the interval remains the same in a linear situation as well as how the interval increases or decreases in a nonlinear situation is developed in high school. Students use recursive reasoning to analyze patterns and structures in tables in order to create functions which model the situation in context.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.F-TF.A. Trigonometric Functions: Extend the domain of trigonometric functions using the unit circle.

Evidence Outcomes

Students Can:

1. (+) Use radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (CCSS: HS.F-TF.A.1)
2. (+) Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. (CCSS: HS.F-TF.A.2)
3. (+) Use special triangles to determine geometrically the values to sine, cosine, tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$ and use the unit circle to express the values sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ and in terms of their values for x where x is any real number. (CCSS: HS.F-TF.A.3)
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. (CCSS: HS.F-TF.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Measure distance around a circle in units the length of the radius of the circle, or radians, and see how this measure stays the same for all equivalent angles, regardless of the circle's size. (MP7)

Inquiry Questions:

1. Trigonometric ratios are defined as ratios of one side of a right triangle to another. What are radians a ratio of?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. Trigonometry is a component of mathematics unique to high school where the functions standard and geometry standard overlap and support each other. In Grade 8, students understand and apply the Pythagorean Theorem.
3. In high school, students begin their study of trigonometry with right triangles. However, this limits the angles considered to those between 0 degrees and 90 degrees. Later, students expand the types of angles considered, and use the unit circle to make connections between the trigonometric ratios derived from right triangles and those of angles not representable by right triangles. Students learn, by similarity, that the radian measure of an angle can be defined as the quotient of arc length to radius. As a quotient of two lengths, therefore, radian measure is "dimensionless," explaining why the "unit" is often omitted when measuring angles in radians.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

HS.F-TF.B. Trigonometric Functions: Model periodic phenomena with trigonometric functions.

Evidence Outcomes

Students Can:

5. (+) Model periodic phenomena with trigonometric functions with specified amplitude, frequency, and midline.★ (CCSS: HS.F-TF.B.5)
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. (CCSS: HS.F-TF.B.6)
7. (+) Use inverse function to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★ (CCSS: HS.F-TF.B.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Students recognize a real-world situation as periodic and construct an appropriate trigonometric representation. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of periodic quantities and their relationships in problem situations, both within real-world contexts and with context removed. (MP2)
3. Apply trigonometric functions and graphs to model periodic situations arising in everyday life, society, and the workplace. (MP4)
4. Use the regularity inherent in periodic functions to gain a deeper understanding of their mathematical characteristics. (MP8)

Inquiry Questions:

1. How does an understanding of the unit circle support an understanding of periodic phenomena?
2. What are examples of phenomena that can be modeled using trigonometric functions?
3. How can you determine if a periodic phenomena should be represented with a sine function or a cosine function?

Coherence Connections:

1. This expectation is in addition to the major work of high school and includes modeling (★) and advanced (+) outcomes.
2. In previous grades, students calculate the area and circumference of circles.
3. In high school, students develop the ideas of periodic motion as simply being the graph of the movement around the circle. Transformations of trigonometric functions should be connected to the structures of transformations for other functions studied in high school.
4. In high school, students apply trigonometry to many different authentic contexts. Of all the subjects that students learn in geometry, trigonometry may have the greatest application in college and careers due in part to its ability to model real-world functions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.F-TF.C. Trigonometric Functions: Prove and apply trigonometric identities.

Evidence Outcomes

Students Can:

8. (+) Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. (CCSS: HS.F-TF.C.8)
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. (CCSS: HS.F-TF.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explain the relationship between algebra and trigonometry. (Civic/Interpersonal Skills: Communication)
2. Make sense of trigonometric quantities as expressions and use their relationships in problem situations. (MP2)
3. See trigonometric expressions as single objects or as being composed of several objects. (MP7)

Inquiry Questions:

1. How is the Pythagorean identity related to the Pythagorean Theorem?
2. How is the identity $\sin^2(\theta) + \cos^2(\theta) = 1$ related to the equation of a circle centered at the origin?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. In Grade 8, students understand and apply the Pythagorean Theorem and its converse.
3. The Pythagorean Identity is a foundational trigonometric identity that must be understood through its components both in and out of context.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

HS.S-ID.A. Interpreting Categorical & Quantitative Data: Summarize, represent, and interpret data on a single count or measurement variable.

Evidence Outcomes

Students Can:

1. Model data in context with plots on the real number line (dot plots, histograms, and box plots). (CCSS: HS.S-ID.A.1)
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (CCSS: HS.S-ID.A.2)
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (CCSS: HS.S-ID.A.3)
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages and identify data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. (CCSS: HS.S-ID.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Understand statistical descriptors of data and interpret and be critical of the use of statistics outside of school. (Professional Skills: Information Literacy)
2. Create, analyze, and synthesize visual representations of statistical data. (Entrepreneurial Skills: Literacy/Reading)
3. Reason about the context of the data separate from the numbers involved and about the numbers separate from the context; move fluidly between contextualized reasoning and decontextualized reasoning. (MP2)
4. Use statistics and statistical reasoning to make sense of, interpret, and generalize about real-world situations. (MP4)

Inquiry Questions:

1. How would you describe the difference between the distributions of two data sets with the same measure of center but different measures of spread?
2. Why do we have multiple measures of center? Why wouldn't we always just use the mean?
3. What questions might a statistician ask about extreme data points? How do they/should they affect the interpretation of the data?

Coherence Connections:

1. This expectation is in addition to the major work of high school.
2. In Grade 6, students study data displays, measures of center, and measures of variability. Standard deviation, introduced in high school, involves much the same principle as the mean absolute deviation (MAD) that students use beginning in Grade 6. Students should see that the standard deviation is the appropriate measure of spread for data distributions that are approximately normal in shape, as the standard deviation then has a clear interpretation related to relative frequency.
3. At this level, students are not expected to fit normal curves to data. Instead, the aim is to look for broad approximations, with application of the rather rough “empirical rule” (also called the 68%–95% Rule) for distributions that are somewhat bell-shaped. The better the bell, the better the approximation.

Prepared Graduates:

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

Grade Level Expectation:

HS.S-ID.B. Interpreting Categorical & Quantitative Data: Summarize, represent, and interpret data on two categorical and quantitative variables.

Evidence Outcomes

Students Can:

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (CCSS: HS.S-ID.B.5)
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. (CCSS: HS.S-ID.B.6)
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (CCSS: HS.S-ID.B.6.a)
   b. Informally assess the fit of a function by plotting and analyzing residuals. (CCSS: HS.S-ID.B.6.b)
   c. Fit a linear function for a scatter plot that suggests a linear association. (CCSS: HS.S-ID.B.6.c)
7. Distinguish between correlation and causation. (CCSS: HS.S-ID.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Create, interpret and demonstrate statistical understanding using technology. (Professional Skills: Use Information and Communications Technologies)
2. Analyze, synthesize, and interpret information from scatter plots and residual plots, and construct explanations for these interpretations. (Entrepreneurial Skills: Literacy/Reading and Writing)
3. Use calculators or computer software to compute with large data sets then interpret and make statistical use of the results. (MP5)
4. Look for patterns in tables and on scatter plots. (MP7)

Inquiry Questions:

1. Does a high correlation (close to ± 1) in the data of two quantitative variables mean that one causes a response in the other? Why or why not?
2. In what way(s) does a plot of the residuals help us consider the best model for a data set?

Coherence Connections:

1. This expectation supports the major work of high school.
2. In Grade 8, students explore scatter plots with linear associations and create equations for informal “lines of best fit” in support of their in-depth study of linear equations. In high school, this statistical topic is formalized and includes fitting quadratic or exponential functions (where appropriate) in addition to linear. Additionally, students use graphing calculators or software to analyze the residuals and interpret the meaning of this analysis in terms of the correctness of fit.
3. It is important that students understand the foundational concept that “correlation does not equal causation” within their study of curve/line-fitting and the associated numerical calculations. This presents a launching point for discussions about the design and analysis of randomized experiments, also included in high school statistics.
4. The mathematics of summarizing, representing, and interpreting data on two categorical or quantitative variables lays the foundation for more advanced statistical topics, such as inference.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP5. Use appropriate tools strategically.

Grade Level Expectation:

HS.S-ID.C. Interpreting Categorical & Quantitative Data: Interpret linear models.

Evidence Outcomes

Students Can:

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (CCSS: HS.S-ID.C.7)
8. Using technology, compute and interpret the correlation coefficient of a linear fit. (CCSS: HS.S-ID.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Critically interpret the use of statistics in their lives outside of school. (Professional Skills: Information Literacy)
2. Use technology and interpret results as they relate to life outside of school. (Professional Skills: Use Information and Communications Technologies)
3. Reason quantitatively about the contextual meaning of slope and intercept of linear models of real-world data, and when the numerical value has no meaning within the context. (MP2)
4. Use technology to compute, model, and reason about linear representations of bivariate data, and interpret the meaning of the calculated values. (MP5)

Inquiry Questions:

1. How is it possible for the intercept of a linear model to not have meaning in the context of the data?
2. What does the correlation coefficient of a linear model tell us? What actions, recommendations, or interpretations might we have about the correlation coefficient?

Coherence Connections:

1. This expectation is in addition to the major work of high school.
2. The comprehensive study of linear functions in Grade 8 allows the high school focus to shift from computation to interpretation of the components of a linear function. Whereas in Grade 8 the slope/rate of change is described mathematically, the work here focuses on the contextual meaning of the rate of change and its applicability to the linear function as a model to predict unknown values of the real-world scenario.
3. The statistics concepts in high school lend themselves to application in other content areas, such as science (e.g., the relationship between cricket chirps and temperature), sports (e.g., the relationship between the year and the average number of home runs in major league baseball), and social studies (e.g., the relationship between returns from buying Treasury bills and from buying common stocks).

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

Grade Level Expectation:

HS.S-IC.A. Making Inferences & Justifying Conclusions: Understand and evaluate random processes underlying statistical experiments.

Evidence Outcomes

Students Can:

1. Describe statistics as a process for making inferences about population parameters based on a random sample from that population. (CCSS: HS.S-IC.A.1)
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability* 0.5. *Would a result of* 5 *tails in a row cause you to question the model?* (CCSS: HS.S-IC.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Students understand how statistics serves to make inferences about a population. (Professional Skills: Information Literacy)
2. Use a variety of statistical tools to construct and defend logical arguments based on data. (MP3)
3. Understand and describe the differences between statistics (derived from samples) and parameters (characteristic of the population). (MP6)

Inquiry Questions:

1. What is the difference between a statistic and a parameter? Why do we need both?
2. Why is it important that random sampling be used to make inferences about population parameters?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grade 7, students approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.
3. The concepts of this expectation are foundational for advanced study of statistical inference.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

HS.S-IC.B. Making Inferences & Justifying Conclusions: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Evidence Outcomes

Students Can:

3. Identify the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (CCSS: HS.S-IC.B.3)
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. (CCSS: HS.S-IC.B.4)
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (CCSS: HS.S-IC.B.5)
6. Evaluate reports based on data. Define and explain the meaning of significance, both statistical (using p-values) and practical (using effect size). (CCSS: HS.S-IC.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Apply statistical methods to interpret information and draw conclusions in real-world contexts. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Evaluate reports based on data and explain the practical and statistical significance of the results. (Entrepreneurial Skills: Literacy/Reading and Writing)
3. Use sampling, design, and results of sample surveys, experiments, and observational studies and justify reasonable responses and misleading or inaccurate results. (MP3)
4. Use sampling, randomization, and simulations to model, describe, and interpret real-world situations, and use margin of error, p-values and effect size to describe the meaning of the results. (MP4)
5. Observe regular patterns in distributions of sample statistics and use them to make generalizations about the population parameter. (MP8)

Inquiry Questions:

1. How can the results of a statistical investigation be used to support or critique a hypothesis?
2. What happens to sample-to-sample variability when you increase the sample size?
3. How does randomization minimize bias?
4. Can the practical significance of a given study matter more than statistical significance? Why is it important to know the difference?
5. Why is the margin of error in a study important?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grades 6–8, students engage with statistics to: (a) draw informal comparative inferences about two populations; (b) informally assess degree of visual overlap of two numerical data distributions; (c) use measures of center and measure of variability for numerical data from random samples to draw comparative inferences; and (d) generate or simulate multiple samples to gauge variation in estimates and predictions. These concepts are extended and formalized in high school.
3. Students' understanding of random sampling is the key that allows the computation of margins of error in estimating a population parameter and can be extended to the random assignment of treatments in an experiment.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP6. Attend to precision.

Grade Level Expectation:

HS.S-CP.A. Conditional Probability & the Rules of Probability: Understand independence and conditional probability and use them to interpret data.

Evidence Outcomes

Students Can:

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). (CCSS: HS.S-CP.A.1)
2. Explain that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS: HS.S-CP.A.2)
3. Using the conditional probability of A given B as $P(A \text{and} B)/P(B)$, interpret the independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (CCSS: HS.S-CP.A.3)
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in 10th grade. Do the same for other subjects and compare the results.* (CCSS: HS.S-CP.A.4)
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* (CCSS: HS.S-CP.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Apply probability concepts and interpret their real-world meaning. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Use probability values to describe how multiple random events are related, including the ideas of independence and conditional probability as they have meaning in the real world. (MP3)
3. Use probability to support the independence of two random events, or to make sense to conditional probabilities. (MP4)
4. Use clear definitions and accurate notation to express probability concepts. (MP6)

Inquiry Questions:

1. How can you describe the formula for determining independence in everyday language? Why does this make sense?
2. How can you describe the formula for conditional probability in everyday language? Why does this make sense?
3. How can a careful and clear display of categorical data in a table help in interpreting relationships between the values expressed?

Coherence Connections:

1. This expectation is in addition to the major work of high school.
2. In Grade 7, students encounter the development of basic probability, including chance processes, probability models, and sample spaces.
3. In high school, the relative frequency approach to probability is extended to conditional probability and independence, rules of probability and their use in finding probabilities of compound events, and the use of probability distributions to solve problems involving expected value. As seen in the expectations for Making Inferences & Justifying Conclusions, there is a strong connection between statistics and probability.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

HS.S-CP.B. Conditional Probability & the Rules of Probability: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Evidence Outcomes

Students Can:

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. (CCSS: HS.S-CP.B.6)
7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. (CCSS: HS.S-CP.B.7)
8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B \mid A) = P(B)P(A \mid B)$, and interpret the answer in terms of the model. (CCSS: HS.S-CP.B.8)
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. (CCSS: HS.S-CP.B.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Understand and apply probability to the real world. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Consider multiple approaches and representations for representing and understanding probabilities of random events. (MP1)
3. Consider probability concepts in context and mathematically and make connections between both types of reasoning. (MP2)
4. Use probability models to represent and make sense of real-world phenomena. (MP4)

Inquiry Questions:

1. What is an everyday situation that helps explain the Addition Rule? How does the context help you understand the subtraction of $P(A \text{ and } B)$ from $P(A) + P(B)$?

Coherence Connections:

1. This expectation is in addition to the major work of high school and includes advanced (+) outcomes.
2. Studying and understanding probability, which is always in a context, provides high school students with a mathematical structure for dealing with the many changes they will experience as part of life.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

HS.S-MD.A. Using Probability to Make Decisions: Calculate expected values and use them to solve problems.

Evidence Outcomes

Students Can:

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. (CCSS: HS.S-MD.A.1)
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. (CCSS: HS.S-MD.A.2)
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* (CCSS: HS.S-MD.A.3)
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* (CCSS: HS.S-MD.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Understand and apply probability to the real world. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Consider probability concepts in context and mathematically, and make connections between both types of reasoning. (MP2)
3. Apply probability models to real-world situations, calculate appropriately, and interpret the results. (MP4)

Inquiry Questions:

1. What is a random variable?
2. Create a context which can be used to describe a random variable.

Coherence Connections:

1. This expectation represents advanced (+) work of high school.

Prepared Graduates:

MP4. Model with mathematics.

Grade Level Expectation:

HS.S-MD.B. Using Probability to Make Decisions: Use probability to evaluate outcomes of decisions.

Evidence Outcomes

Students Can:

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. (CCSS: HS.S-MD.B.5)
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or game at a fast-food restaurant.* (CCSS: HS.S-MD.B.5.a)
   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or major accident.* (CCSS: HS.S-MD.B.5.b)
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). (CCSS: HS.S-MD.B.6)
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). (CCSS: HS.S-MD.B.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Students understand and apply probability to the real world. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Apply probability models to real-world situations, calculate appropriately, and interpret the results. (MP4)

Inquiry Questions:

1. How does probability help in the decision-making process?
2. Why does expected value require the weighted average of all possible values?

Coherence Connections:

1. The expectation represents advanced (+) work of high school.

Prepared Graduates:

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

Grade Level Expectation:

HS.G-CO.A. Congruence: Experiment with transformations in the plane.

Evidence Outcomes

Students Can:

1. State precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (CCSS: HS.G-CO.A.1)
2. Represent transformations in the plane using e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (CCSS: HS.G-CO.A.2)
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (CCSS: HS.G-CO.A.3)
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (CCSS: HS.G-CO.A.4)
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using appropriate tools (e.g., graph paper, tracing paper, or geometry software). Specify a sequence of transformations that will carry a given figure onto another. (CCSS: HS.G-CO.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explore transformations in the plane using concrete and technological tools. The use of tools allows students to attend to precision. (MP5)
2. Use exact terms, symbols and notation when describing and working with geometric transformations. (MP6)

Inquiry Questions:

1. What is the relationship between functions and geometric transformations?
2. How is a figure's symmetry connected to congruence transformations?

Coherence Connections:

1. This expectation supports the major work of high school.
2. In Grade 8, rotations, reflections, and translations are developed experimentally and students graph points in all four quadrants of the coordinate plane.
3. In high school, transformations are studied in terms of functions, where the inputs and outputs are points in the coordinate plane, and students understand the meaning of rotations, reflections, and translations based on angles, circles, perpendicular lines, parallel lines, and line segments.
4. Geometric reasoning is expressed through formal proof and precise language, informal explanation and construction, and strategic experimentation to verify or refute claims.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

Grade Level Expectation:

HS.G-CO.B. Congruence: Understand congruence in terms of rigid motions.

Evidence Outcomes

Students Can:

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (CCSS: HS.G-CO.B.6)
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (CCSS: HS.G-CO.B.7)
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (CCSS: HS.G-CO.B.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Justify claims of congruence in terms of rigid motions and follow others' reasoning in describing alternate rigid motions that lead to the same congruence conclusion. (MP3)
2. Examine claims and make explicit use of definitions to support formal proof and justification of congruence relationships. (MP6)

Inquiry Questions:

1. How can transformations be used to prove that two triangles are congruent?
2. What is the minimum amount of information you need to know about two triangles in order to determine if they are congruent? Why is that the minimum?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grade 8, students study rigid motions using physical models or software, with an emphasis on geometric intuition, whereas high school geometry weighs precise definitions and geometric intuition equally.
3. In high school, students will compare graphs of functions and other curves to make congruence and similarity arguments based on rigid motions.

COLORADO
Department of Education

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

Grade Level Expectation:

HS.G-CO.C. Congruence: Prove geometric theorems.

Evidence Outcomes

Students Can:

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* (CCSS: HS.G-CO.C.9)
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to* 180°*; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* (CCSS: HS.G-CO.C.10)
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* (CCSS: HS.G-CO.C.11)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Justify claims of congruence in terms of rigid motions, understand alternate reasoning, and recognize and address errors when appropriate. (MP3)
2. Make explicit use of definitions, symbols, and notation with lines, angles, triangles, and parallelograms. (MP6)

Inquiry Questions:

1. Can some theorems be proved without using other, previously proven theorems? If so, what does that imply about a system of theorems?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grades 7 and 8, students investigate properties of lines and angles, triangles, and parallelograms.
3. In high school, proof is sometimes formatted with a two-column approach, with one column headed "statements" and the other column headed "reasons." Students may also write sentences (paragraph proof), or use boxes (flow proof), or they may employ other formats, or combine formats, for communicating proof.

Colorado Academic Standards
Mathematics

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

Grade Level Expectation:

HS.G-CO.D. Congruence: Make geometric constructions.

Evidence Outcomes

Students Can:

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* (CCSS: HS.G-CO.D.12)
13. (+) Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. (CCSS: HS.G-CO.D.13)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use a variety of tools, appropriate to the task, to make geometric constructions. (MP5)
2. Precisely use construction tools and communicate their steps, reasoning, and results using mathematical language. (MP6)

Inquiry Questions:

1. How is a geometric construction like a proof?
2. How can you use properties of circles to ensure precise constructions?

Coherence Connections:

1. This expectation supports the major work of high school and includes an advanced (+) outcome.
2. In Grade 7, students draw geometric shapes with rulers, protractors, and technology.
3. In high school, students use proofs to justify validity of their constructions. They use geometric constructions to precisely locate the line of reflection between an image and its pre-image and to accurately draw a figure under a translation or rotation and justify its validity.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

HS.G-SRT.A. Similarity, Right Triangles, and Trigonometry: Understand similarity in terms of similarity transformations.

Evidence Outcomes

Students Can:

1. Verify experimentally the properties of dilations given by a center and a scale factor. (CCSS: HS.G-SRT.A.1)
   a. Show that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. (CCSS: HS.G-SRT.A.1.a)
   b. Show that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. (CCSS: HS.G-SRT.A.1.b)
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (CCSS: HS.G-SRT.A.2)
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (CCSS: HS.G-SRT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use auxiliary lines not part of the original figure when reasoning about similarity. (MP2)
2. Employ geometric tools and technology (including dynamic geometric software) in exploring and verifying the properties of dilations and in understanding the properties of similar figures. (MP5)
3. Connect algebraic and geometric content when using proportional reasoning to determine if two figures are similar. (MP7)
4. Recognize and use repeated reasoning in exploring and verifying the properties of dilations and similarity and in establishing the AA criterion for similar triangles. (MP8)

Inquiry Questions:

1. How can we use the concepts of similarity to measure real-world objects that are difficult or impossible to measure directly?
2. How are similarity and congruence related to one another?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grade 8, students informally investigate dilations and similarity, including the AA criterion.
3. In high school, students show that two figures are similar by finding a scaling transformation (dilation or composition of dilation with a rigid motion) or a sequence of scaling transformations that maps one figure to the other, and recognize that congruence is a special case of similarity where the scale factor is equal to 1.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP6. Attend to precision.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

HS.G-SRT.B. Similarity, Right Triangles, and Trigonometry: Prove theorems involving similarity.

Evidence Outcomes

Students Can:

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* (CCSS: HS.G-SRT.B.4)
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (CCSS: HS.G-SRT.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Justify reasoning using logical, cohesive steps when proving theorems and solving problems in geometry. (MP3)
2. Use precise geometric and other mathematical terms and symbols to construct proofs and solve problems in geometry. (MP6)
3. Maintain oversight of the problem-solving process and when writing proofs while attending to details and continually evaluating the reasonableness of intermediate results. (MP8)

Inquiry Questions:

1. How does the Pythagorean Theorem support the case for triangle similarity?

Coherence Connections:

1. This expectation represents major work of high school.
2. In Grade 7, students study proportional relationships and apply them to solve real-world problems.
3. In Grade 8, students are introduced to the concept of similar figures using physical models and geometry software.
4. In high school, students understand properties of similar triangles to develop understanding of right triangle trigonometry.

COLORADO
Department of Education

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.G-SRT.C. Similarity, Right Triangles, and Trigonometry: Define trigonometric ratios and solve problems involving right triangles.

Evidence Outcomes

Students Can:

6. Explain that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (CCSS: HS.G-SRT.C.6)
7. Explain and use the relationship between the sine and cosine of complementary angles. (CCSS: HS.G-SRT.C.7)
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ (CCSS: HS.G-SRT.C.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason abstractly by relating the properties of similar triangles to the definitions of the trigonometric ratios for acute angles, recognizing that the proportionality of side measures creates a single ratio based on the angle measure, regardless of the size of the right triangle. (MP2)
2. Apply trigonometric ratios and the Pythagorean Theorem to model and solve real-world problems. (MP4)
3. Use structure to relate triangle similarity and the trigonometric ratios for acute angles. (MP7)

Inquiry Questions:

1. How are the trigonometric ratios for acute angles connected to the properties of similar triangles?
2. What visual representation(s) explains why the sine of an acute angle is equivalent to the cosine of its complement?

Coherence Connections:

1. This expectation represents major work of high school and includes a modeling (★) outcome.
2. In Grade 7, students apply proportional reasoning and solve problems involving scale drawings of geometric figures. In Grade 8, students connect proportional relationships to triangles representing the slope of a line and understand congruence and similarity using physical models, transparencies, or geometry software.
3. In high school, students apply their previous study of similarity to establish understanding of the trigonometric ratios for acute angles. They connect right triangle trigonometry to concepts with algebra and functions. They understand that trigonometric ratios are functions of the size of an angle, and use the Pythagorean Theorem to show that $\sin^2(\theta) + \cos^2(\theta) = 1$.

COLORADO Department of Education

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP3. Construct viable arguments and critique the reasoning of others.

Grade Level Expectation:

HS.G-SRT.D. Similarity, Right Triangles, and Trigonometry: Apply trigonometry to general triangles.

Evidence Outcomes

Students Can:

9. (+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (CCSS: HS.G-SRT.D.9)
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems. (CCSS: HS.G-SRT.D.10)
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (CCSS: HS.G-SRT.D.11)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of the ambiguous case of the Law of Sines and persevere in determining valid and invalid solutions. (MP1)
2. Construct an argument proving the Laws of Sines and Cosines. (MP3)

Inquiry Questions:

1. Why does the formula $A = \frac{1}{2}ab\sin(C)$ accurately calculate the area of a triangle?
2. In using the Law of Sines, when do we need to consider the ambiguous case?
3. How is the Law of Cosines related to the Pythagorean Theorem?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. In Grade 6, students learn to calculate the area of a triangle by developing the formula $A = \frac{1}{2}bh$. In Grade 8, students understand and apply the Pythagorean Theorem.
3. In high school, students develop understanding of right triangle trigonometry through similarity. In advanced courses, students prove trigonometric identities using relationships between sine and cosine.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.G-C.A. Circles: Understand and apply theorems about circles.

Evidence Outcomes

Students Can:

1. Prove that all circles are similar. (CCSS: HS.G-C.A.1)
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* (CCSS: HS.G-C.A.2)
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (CCSS: HS.G-C.A.3)
4. (+) Construct a tangent line from a point outside a given circle to the circle. (CCSS: HS.G-C.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Justify reasoning and use logical, cohesive steps when proving theorems and solving problems in geometry. (MP3)
2. Employ geometric tools and technology (including dynamic geometry software) in exploring relationships in circles and in circle-related constructions. (MP5)
3. Observe the relationships among angles in circles and extend their conclusions to a variety of scenarios. (MP7)

Inquiry Questions:

1. Draw or find examples of several different circles. In what ways are they related geometrically? How can you describe these relationships in terms of transformations?

Coherence Connections:

1. This expectation supports the major work of high school and includes an advanced (+) outcome.
2. In Grade 7, students informally derive and apply the equations of area and circumference of circles.
3. In high school, students relate circle properties to geometric constructions and proofs of their validity.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.G-C.B. Circles: Find arc lengths and areas of sectors of circles.

Evidence Outcomes

Students Can:

5. (+) Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (CCSS: HS.G-C.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use verbal and written arguments using similarity to justify arc lengths and radian measures. (MP3)
2. Attend to precise mathematical definitions, relationships, and symbols to describe and solve problems involving arc lengths and areas of sectors of circles. (MP6)
3. Use understanding of the area of a circle and the meaning of a central angle to synthesize the formula for the area of a sector. (MP7)

Inquiry Questions:

1. In what ways is it more convenient to use radian measure for a central angle in a circle, rather than degree measure?

Coherence Connections:

1. This expectation represents advanced (+) work of high school.
2. In Grade 7, students know the formulas for area and circumference of a circle and apply proportional reasoning to real-world problems.
3. In high school, the formulas for area and circumference of a circle are generalized to fractional parts of a circle. Students apply proportional reasoning to find the length of an arc of a circle.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.G-GPE.A. Expressing Geometric Properties with Equations: Translate between the geometric description and the equation for a conic section.

Evidence Outcomes

Students Can:

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (CCSS: HS.G-GPE.A.1)
2. (+) Derive the equation of a parabola given a focus and directrix. (CCSS: HS.G-GPE.A.2)
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. (CCSS: HS.G-GPE.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make conjectures about the form and meaning of an equation for a conic section, and plan a solution pathway that deliberately connects the geometric and algebraic representations of conic sections rather than simply jumping into a solution attempt. (MP1)
2. Use abstract and quantitative reasoning to apply the Pythagorean Theorem to the equations of conic sections, particularly circles and parabolas. (MP2)
3. Analyze the underlying structure of the equations for conic sections and their connection to the Pythagorean Theorem and to each other. (MP7)

Inquiry Questions:

1. How does the Pythagorean Theorem connect to the general equation for a circle with center (a, b) and radius r? How can this be illustrated with a diagram?
2. How does the Pythagorean Theorem connect to the equation for a parabola? How can this be illustrated with a diagram?

Coherence Connections:

1. This expectation is in addition to the major work of high school and includes advanced (+) outcomes.
2. In Grade 8, students apply the Pythagorean theorem to find the length of an unknown side of a right triangle and calculate the distance between two points in the coordinate plane.
3. In high school, the application of the Pythagorean theorem is generalized to obtain formulas related to conic sections. Quadratic functions and completing the square are studied in the domain of interpreting functions. The methods are applied here to transform a quadratic equation representing a conic section into standard form.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

HS.G-GPE.B. Expressing Geometric Properties with Equations: Use coordinates to prove simple geometric theorems algebraically.

Evidence Outcomes

Students Can:

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0,2)$. (CCSS: HS.G-GPE.B.4)
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (CCSS: HS.G-GPE.B.5)
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (CCSS: HS.G-GPE.B.6)
7. Use coordinates and the distance formula to compute perimeters of polygons and areas of triangles and rectangles.★ (CCSS: HS.G-GPE.B.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Connect coordinate proof to geometric theorems and the coordinate plane. (MP2)
2. Justify theorems involving distance and ratio, both verbally and written. (MP3)
3. Apply understandings of distance and perpendicularity to polygons. (MP7)

Inquiry Questions:

1. What mathematical concepts and tools become available when coordinates are applied to geometric figures?

Coherence Connections:

1. This expectation represents major work of high school and includes a modeling (★) outcome.
2. In Grade 8, students relate the slope triangles of a line to proportions and similarity, and they apply the Pythagorean theorem to determine distances in the coordinate plane.
3. In high school, students prove theorems using coordinates, and they use algebraic and geometric concepts to connect the equations of conic sections and their corresponding graphs.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

HS.G-GMD.A. Geometric Measurement and Dimension: Explain volume formulas and use them to solve problems.

Evidence Outcomes

Students Can:

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* (CCSS: HS.G-GMD.A.1)
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. (CCSS: HS.G-GMD.A.2)
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ (CCSS: HS.G-GMD.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Formulate justifications of the formulas for circumference of a circle, area of a circle, and volumes of cylinders, pyramids, and cones. (MP3)
2. Apply volume formulas for cylinders, pyramids, cones, and spheres to real-world contexts to solve problems. (MP4)
3. Apply technologies, as appropriate, to estimate and compute areas and volumes. (MP5)

Inquiry Questions:

1. How could you use other geometric relationships to explain why the volume of a cylinder is $V = \pi r^2 h$?
2. How could you algebraically prove that a right cylinder and a corresponding oblique cylinder have the same volume?

Coherence Connections:

1. This expectation is in addition to the major work of high school and includes modeling (★) and advanced (+) outcomes.
2. In Grade 7, students informally derive the formula for the area of a circle from the circumference. In Grade 8, students know and use the formulas for volumes of cylinders, cones, and spheres.
3. In high school, students construct informal justifications of volume formulas. Students might view a pyramid as a stack of layers and, using Cavalieri's Principle, see that shifting the layers does not change the volume. Furthermore, stretching the height of the pyramid by a given scale factor thickens each layer by the scale factor which multiplies its volume by that factor. Using such arguments, students can derive the formula for the volume of any pyramid with a square base.
4. Reasoning about area and volume geometrically prepares students for topics in calculus.

COLORADO Department of Education

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

Grade Level Expectation:

HS.G-GMD.B. Geometric Measurement and Dimension: Visualize relationships between two-dimensional and three-dimensional objects.

Evidence Outcomes

Students Can:

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (CCSS: HS.G-GMD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Conceptualize problems using concrete objects or pictures, checking answers using different methods, and continually asking themselves, “Does this make sense?” (MP1)
2. Reason abstractly to visualize, describe, and justify their understanding of cross-sections of three-dimensional objects and of three-dimensional objects generated by rotations of two-dimensional objects without concrete representations. (MP2)

Inquiry Questions:

1. When will the shape of a cross-section of a three-dimensional object be the same for all planes that intersect the object? How do you know?

Coherence Connections:

1. This expectation supports the major work of high school.
2. In Grades 6–8, students apply geometric measurement to real-world and mathematical problems, making use of properties of figures as they dissect and rearrange them in order to calculate or estimate lengths, areas, and volumes.
3. In high school, students examine geometric measurement more closely, giving informal arguments to explain formulas, drawing on abilities developed in earlier grades: dissecting and rearranging two- and three-dimensional figures; and visualizing cross-sections of three-dimensional figures.
4. In calculus, students use integrals to calculate the volume of solids formed by rotating a curve around an axis.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

HS.G-MG.A. Modeling with Geometry: Apply geometric concepts in modeling situations.

Evidence Outcomes

Students Can:

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★ (CCSS: HS.G-MG.A.1)
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★ (CCSS: HS.G-MG.A.2)
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ (CCSS: HS.G-MG.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of real-world shapes and spaces by applying geometric concepts. (MP1)
2. Apply the properties and relationships associated with geometric figures and measurement to make sense of, reason about, and solve real-world problems. (MP4)
3. Model and solve problems involving geometric figures and measurement using technology and dynamic geometry software. (MP5)

Inquiry Questions:

1. What are all the ways you would use geometry to design a figure in 3D software, estimate the mass of printer filament needed to 3D print a $\frac{1}{8}$-scale model of your figure, then calculate the cost to produce the figure out of a different material at full size?

Coherence Connections:

1. This expectation represents major work of high school and includes modeling (★) outcomes.
2. In high school, geometric modeling can be used in Fermi problems, problems which ask for rough estimates of quantities and often involve estimates of densities.
3. In high school, students apply trigonometric measurement to many different authentic contexts. Of all the subjects students learn in geometry, trigonometry may have the greatest application in college and careers. Applying abstract geometric concepts involving congruence, similarity, measurement, trigonometry, and other related areas to solving problems situated in real-world contexts provides a means of building understanding about concepts and experiencing the usefulness of geometry.

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

	Unknown Product	Group Size Unknown ("How many in each group? Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red had costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long as the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first.
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here, a, b, and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every a there exists $-a$ so that $a+\ (-a)=\ (-a)+a=0$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Table 4. The properties of equality. Here, a, b, and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a=b$, then $b=a$.
Transitive property of equality	If $a=b$ and $b=c$, then $a=c$.
Addition property of equality	If $a=b$, then $a+c=b+c$.
Subtraction property of equality	If $a=b$, then $a-c=b-c$.
Multiplication property of equality	If $a=b$, then $a\times c=b\times c$.
Division property of equality	If $a=b$ and $c\neq 0$, then $a\div c=b\div c$.
,Substitution property of equality	If $a=b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here, a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a<b$, $a=b$, $a>b$.

If $a>b$ and $b>c$ then $a>c$.

If $a>b$, then $b<a$.

If $a>b$, then $-a<-b$.

If $a>b$, then $a\pm c>b\pm c$.

If $a>b$ and $c>0$, then $a\times c>b\times c$.

If $a>b$ and $c<0$, then $a\times c<b\times c$.

If $a>b$ and $c>0$, then $a\div c>b\div c$.

If $a>b$ and $c<0$, then $a\div c<b\div c$.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Some examples of situations requiring modeling might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and financial investments.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.