Colorado
Academic Standards

Mathematics

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards:

MP1. Make sense of problems and persevere in solving them.

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

MP6. Attend to precision.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

COLORADO
Department of Education

Prepared Graduates:

MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.NS.A. The Number System: Know that there are numbers that are not rational, and approximate them by rational numbers.

Evidence Outcomes

Students Can:

1. Demonstrate informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. Define irrational numbers as numbers that are not rational. (CCSS: 8.NS.A.1)
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS: 8.NS.A.2)
3. Apply understanding of rational and irrational numbers to describe and work within the structure of the real number system effectively and efficiently. (MP7)
4. Recognize repetition in decimal expansions of rational numbers and recognize when a decimal expansion cannot be represented by a rational number. (MP8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Investigate rational and irrational numbers and their relative approximate positions on a number line. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use technology to look for repetition in decimal expansions and use number lines to order and compare irrational numbers relative to rational numbers. (MP5)

Inquiry Questions:

1. How many irrational numbers exist?
2. Why is there no real number closest to zero?
3. Can you accurately plot an irrational number on the number line? How do you know?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 7, students apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
3. In Grade 8, this expectation supports working with radicals and integer exponents. This concludes the introduction of all numbers that comprise the real number system.
4. In high school, students use properties of rational and irrational numbers, work with rational exponents, and extend their understanding of number systems to include complex numbers.

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Prepared Graduates:

MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.EE.A. Expressions & Equations: Work with radicals and integer exponents.

Evidence Outcomes

Students Can:

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (CCSS: 8.EE.A.1)
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares (up to 100) and cube roots of small perfect cubes (up to 64). Know that $\sqrt{2}$ is irrational. (CCSS: 8.EE.A.2)
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3 *times* 10^8 *and the population of the world as* 7 *times* 10^9*, and determine that the world population is more than* 20 *times larger.* (CCSS: 8.EE.A.3)
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (CCSS: 8.EE.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason about unusually large or small quantities. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use calculators or computers to compute with and approximate radicals and roots, and understand how such tools represent scientific notation. (MP5)
3. Explore the structure of numerical expressions with integer exponents to generate equivalent expressions. (MP7)
4. Look for how positive integer exponents are equivalent to repeated multiplication by the base and how negative integer exponents are equivalent to repeated division by the base. (MP8)

Inquiry Questions:

1. How is performing operations on numbers in scientific notation similar to or different from performing operations on numbers in standard notation?
2. Why does a positive number raised to a negative exponent not equal a negative number?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 5, students use whole-number exponents to denote powers of ten. In Grades 6 and 7, they work with algebraic and numerical expressions containing whole-number exponents.
3. In Grade 8, this expectation connects with knowing that there are numbers that are not rational and approximating them by rational numbers, understanding and applying the Pythagorean Theorem, and solving real-world and mathematical problems involving volume of cylinders, cones, and spheres. In high school, students extend work with exponents to rational exponents.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.EE.B. Expressions & Equations: Understand the connections between proportional relationships, lines, and linear equations.

Evidence Outcomes

Students Can:

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (CCSS: 8.EE.B.5)
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (CCSS: 8.EE.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make connections between representations of linear growth. (Entrepreneurial Skills: Inquiry/Analysis)
2. Record information about constant rates of change in graphs and equations. (Entrepreneurial Skills: Literacy/Writing)
3. Make sense of and compare proportional relationships represented in different forms. (MP1)
4. Compare, contrast, and make claims with proportional relationships based on properties of equations, tables, and/or graphs. (MP3)
5. Explore the structure of proportional relationships expressed as equations or graphs for methods of comparison. (MP7)

Inquiry Questions:

1. How is the unit rate of a proportional relationship related to the slope of its graphical representation?
2. Why are similar triangles effective for describing slope geometrically?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students recognize and represent proportional relationships, calculate the constant of proportionality, and graph proportional relationships on the coordinate plane, recognizing that they always pass through the origin.
3. In Grade 8, this expectation connects with analyzing and solving linear equations and pairs of simultaneous linear equations, with defining, evaluating, and comparing functions, and with understanding congruence and similarity using physical models, transparencies, or geometry software.
4. In high school, students compare multiple representations of inverse proportional relationships.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.EE.C. Expressions & Equations: Analyze and solve linear equations and pairs of simultaneous linear equations.

Evidence Outcomes

Students Can:

7. Solve linear equations in one variable. (CCSS: 8.EE.C.7)
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (CCSS: 8.EE.C.7.a)
   b. Solve linear equations with rational number coefficients, including equations with variables on both sides and whose solutions require expanding expressions using the distributive property and collecting like terms. (CCSS: 8.EE.C.7.b)
8. Analyze and solve pairs of simultaneous linear equations. (CCSS: 8.EE.C.8)
   a. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. (CCSS: 8.EE.C.8.a)
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be* 5 *and* 6. (CCSS: 8.EE.C.8.b)
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (CCSS: 8.EE.C.8.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving linear equations and systems of linear equations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Solve problems that require a system of linear equations in two variables. (MP1)
3. Model real-world problems with linear equations and systems of linear equations, with variables defined in their real-world context. (MP4)
4. Solve equations and systems of equations and express solutions with accuracy that makes sense in the real-world context modeled by the equations. (MP6)
5. Recognize the structure of equations and of systems of equations that produce one, infinitely many, or no solution. (MP7)

Inquiry Questions:

1. What is meant by a “solution” to a linear equation? What is meant by a “solution” to a system of two linear equations? How are these concepts related?
2. How is it possible for an equation to have more than one solution? How is it possible for an equation to have no solution?
3. Why can’t a system of linear equations have a solution set other than one, zero, or infinitely many solutions?
4. What connections exist between the graphical solution and the algebraic solution of a system of linear equations?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students reason about and solve one-step and two-step, one-variable equations and inequalities, use properties of operations to generate equivalent expressions, and solve real-world and mathematical problems using numerical and algebraic expressions and equations.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations and with investigating patterns of association in bivariate data.
4. In high school, students abstract and generalize about linear functions and how they compare and contrast to nonlinear functions. Students also reason about and solve systems of equations that include one or more nonlinear equations.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.F.A. Functions: Define, evaluate, and compare functions.

Evidence Outcomes

Students Can:

1. Define a function as a rule that assigns to each input exactly one output. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required for Grade 8.) (CCSS: 8.F.A.1)
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (CCSS: 8.F.A.2)
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$, $(2,4)$ and $(3,9)$, which are not on a straight line.* (CCSS: 8.F.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make connections between the information gathered through tables, equations, graphs, and verbal descriptions of functions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Define variables as quantities and interpret ordered pairs from a functional relationship with respect to those variables. (MP2)
3. With and without technology, analyze and describe functions that are not linear with the use of equations, graphs, and tables. (MP5)
4. See a function as a rule that assigns each input to exactly one output; this structure does not "turn inputs into outputs"; rather, it describes the relationship between items in two sets. (MP7)
5. Recognize patterns of linear growth in different representations of linear functions. (MP8)

Inquiry Questions:

1. Why is it important to know if a mathematical relationship is a function?
2. How can you determine if a function is linear or nonlinear?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations and with using functions to model relationships between quantities.
4. In high school, students use function notation, analyze functions using different representations, build new functions from existing functions, and extend from linear functions to quadratic, exponential, and other more advanced functions.

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Prepared Graduates:

MP4. Model with mathematics.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.F.B. Functions: Use functions to model relationships between quantities.

Evidence Outcomes

Students Can:

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS: 8.F.B.4)
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS: 8.F.B.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Describe in writing the qualitative features of linear or nonlinear functions. (Entrepreneurial Skills: Literacy/Writing)
2. Model real-world situations with linear functions. (MP4)
3. Explore properties of linear functions and how those properties appear in the structure of linear equations in slope-intercept form. (MP7)
4. Use strategies to calculate the rate of change in a linear function (slope) and use properties of linear functions to create equations. (MP8)

Inquiry Questions:

1. What is the minimum information needed to write a linear function for a relationship between two quantities?
2. What are some quantitative and qualitative features of graphs of functions?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 7, students analyze proportional relationships and use them to solve real-world and mathematical problems.
3. In Grade 8, this expectation connects with defining, evaluating, and comparing functions and with investigating patterns of association in bivariate data.
4. In high school, students use function notation, analyze functions using different representations, build new functions from existing functions, and extend from linear functions to quadratic, exponential, and other more advanced functions.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

8.SP.A. Statistics & Probability: Investigate patterns of association in bivariate data.

Evidence Outcomes

Students Can:

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (CCSS: 8.SP.A.1)
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (CCSS: 8.SP.A.2)
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of* 1.5 *cm/hr as meaning that an additional hour of sunlight each day is associated with an additional* 1.5 *cm in mature plant height.* (CCSS: 8.SP.A.3)
4. Explain that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* (CCSS: 8.SP.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Recognize and describe patterns in bivariate data. (Entrepreneurial Skills: Inquiry/Analysis)
2. Interpret the contextual meaning of slope and y-intercept, where applicable in a linear model fit to bivariate data. (MP2)
3. Build statistical models to explore, describe, and generalize the relationship between two variables. (MP4)
4. Use scatter plots and two-way tables to describe possible associations in bivariate data. (MP7)

Inquiry Questions:

1. In what ways is a scatter plot useful in describing and interpreting the relationship between two quantities?
2. Why would we create a linear model for a set of bivariate data?
3. How do you know when a credible prediction can be made from a linear model of bivariate data?
4. What does a pattern of association look like for categorical data?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In previous grades, students apply and extend previous understandings of numbers to the system of rational numbers.
3. In Grade 8, this expectation supports using functions to model relationships between quantities.
4. In high school, students summarize, represent, and interpret data on two categorical and quantitative variables, interpret linear models, and understand independence and conditional probability.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.G.A. Geometry: Understand congruence and similarity using physical models, transparencies, or geometry software.

Evidence Outcomes

Students Can:

1. Verify experimentally the properties of rotations, reflections, and translations: (CCSS: 8.G.A.1)
   a. Lines are taken to lines, and line segments to line segments of the same length. (CCSS: 8.G.A.1.a)
   b. Angles are taken to angles of the same measure. (CCSS: 8.G.A.1.b)
   c. Parallel lines are taken to parallel lines. (CCSS: 8.G.A.1.c)
2. Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (CCSS: 8.G.A.2)
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (CCSS: 8.G.A.3)
4. Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (CCSS: 8.G.A.4)
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* (CCSS: 8.G.A.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Think about how rotations, reflections, and translations of a geometric figure preserve congruence as similar to how properties of operations such as the associative, commutative, and distributive properties preserve equivalence of arithmetic and algebraic expressions. (Entrepreneurial Skills: Critical Thinking/Problem Solving and Inquiry/Analysis)
2. Explain a sequence of transformations that results in a congruent or similar triangle. (MP3)
3. Use physical models, transparencies, geometric software, or other appropriate tools to explore the relationships between transformations and congruence and similarity. (MP5)
4. Use the structure of the coordinate system to describe the locations of figures obtained with rotations, reflections, and translations. (MP7)
5. Reason that since any one rotation, reflection, or translation of a figure preserves congruence, then any sequence of those transformations must also preserve congruence. (MP8)

Inquiry Questions:

1. How are properties of rotations, reflections, translations, and dilations connected to congruence?
2. How are properties of rotations, reflections, translations, and dilations connected to similarity?
3. Why are angle measures significant regarding the similarity of two figures?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students solve problems involving angle measure, area, surface area, and volume, and draw, construct, and also describe geometrical figures and the relationships between them.
3. In Grade 8, this expectation connects with understanding the connections between proportional relationships, lines, and linear equations.
4. In high school, students extend their work with transformations, apply the concepts of transformations to prove geometric theorems, and use similarity to define trigonometric functions.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

8.G.B. Geometry: Understand and apply the Pythagorean Theorem.

Evidence Outcomes

Students Can:

6. Explain a proof of the Pythagorean Theorem and its converse. (CCSS: 8.G.B.6)
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS: 8.G.B.7)
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS: 8.G.B.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Think of the Pythagorean Theorem as not just a formula, but a formula that only holds true under certain conditions. (Entrepreneurial Skills: Inquiry/Analysis)
2. Construct a viable argument about why a proof of the Pythagorean Theorem is valid. (MP3)
3. Test to see if a triangle is a right triangle by applying the Pythagorean Theorem. (MP7)
4. Use patterns to recognize and generate Pythagorean triples. (MP8)

Inquiry Questions:

1. What is the relationship between the Pythagorean Theorem and its converse? In what ways is each useful?
2. Is it always possible to use the Pythagorean Theorem to find the distance between points on the coordinate plane? How do you know?

Coherence Connections:

1. This GLE represents major work of the grade.
2. In Grades 6 and 7, students solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
3. In Grade 8, this expectation connects with radicals and integer exponents, square roots, and solving simple equations in the form $x^2 = p$.
4. In high school, students (a) prove and apply trigonometric identities, (b) prove theorems involving similarity, (c) define trigonometric ratios and solve problems involving right triangles, (d) translate between the geometric description and the equation for a conic section, and (e) use coordinates to prove simple geometric theorems algebraically.

COLORADO
Department of Education

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.

Grade Level Expectation:

8.G.C. Geometry: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Evidence Outcomes

Students Can:

9. State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. (CCSS: 8.G.C.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Efficiently solve problems using established volume formulas. (Professional Skills: Task/Time Management)
2. Describe how the formulas for volumes of cones, cylinders, and spheres relate to one another and to the volume formulas for solids with rectangular bases. (MP3)
3. Use appropriate precision when solving problems involving measurements and volume formulas that describe real-world shapes. (MP6)

Inquiry Questions:

1. How are the formulas of cones, cylinders, and spheres similar to each other?
2. How are the formulas of cones, cylinder, and spheres connected to the formulas for pyramids, prisms, and cubes?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 7, students solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
3. In Grade 8, this expectation connects with radicals and integer exponents.
4. In high school, students apply geometric concepts in mathematical modeling situations and to solve design problems.