Colorado Academic Standards

Mathematics

Third Grade – Fifth Grade

COLORADO
Department of Education

ALL STUDENTS • ALL STANDARDS

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Purpose of Mathematics

"Pure mathematics is, in its way, the poetry of logical ideas."

~Albert Einstein, *Obituary for Emmy Noether* (1935)

"Systematization is a great virtue of mathematics, and if possible, the student has to learn this virtue, too. But then I mean the activity of systematizing, not its result. Its result is a system, a beautiful closed system, closed with no entrance and no exit. In its highest perfection it can even be handled by a machine. But for what can be performed by machines, we need no humans. What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics."

~Hans Freudenthal, *Why to Teach Mathematics So as to Be Useful* (1968)

Mathematics is the human activity of reasoning with number and shape, in concert with the logical and symbolic artifacts that people develop and apply in their mathematical activity. The National Council of Teachers of Mathematics (2018) outlines three primary purposes for learning mathematics:

1. To Expand Professional Opportunity. Just as the ability to read and write was critical for workers when the early 20th century economy shifted from agriculture to manufacturing, the ability to do mathematics is critical for workers in the 21st-century as the economy has shifted from manufacturing to information technology. Workers with a robust understanding of mathematics are in demand by employers, and job growth in STEM (science, technology, engineering, and mathematics) fields is forecast to accelerate over the next decade.

2. Understand and Critique the World. A consequence of living in a technological society is the need to interpret and understand the mathematics behind our social, scientific, commercial, and political systems. Much of this mathematics appears in the way of statistics, tables, and graphs, but this need to understand and critique the world extends to the application of mathematical models, attention given to precision, bias in data collection, and the soundness of mathematical claims and arguments. Learners of mathematics should feel empowered to make sense of the world around them and to better participate as an informed member of a democratic society.

3. Experience Wonder, Joy, and Beauty. Just as human forms and movement can be beautiful in dance, or sounds can make beautiful music, the patterns, shapes, and reasoning of mathematics can also be beautiful. On a personal level, mathematical problem solving can be an authentic act of individual creativity, while on a societal level, mathematics both informs and is informed by the culture of those who use and develop it, just as art or language is used and developed.

References

National Council of Teachers of Mathematics (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: National Council of Teachers of Mathematics.

Prepared Graduates in Mathematics

Prepared graduates in mathematics are described by the eight *Standards for Mathematical Practice* described in the Common Core State Standards (CCSSI, 2010). Across the curriculum at every grade, students are expected to consistently have opportunities to engage in each of the eight practices. The practices aligned with each Grade Level Expectation in the Colorado Academic Standards represent the *strongest potential* alignments between content and the practices, and are not meant to exclude students from engaging in the rest of the practices.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

Math Practice MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Math Practice MP2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Math Practice MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Math Practice MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Math Practice MP5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Math Practice MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Math Practice MP7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Math Practice MP8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1,2)$ with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

References

Common Core State Standards Initiative. (2010). *Standards for mathematical practice*. *http://www.corestandards.org/Math/Practice*

Standards in Mathematics

The Colorado Academic Standards in mathematics are the topical organization of the concepts and skills every Colorado student should know and be able to do throughout their preschool through twelfth grade experience. The standards of mathematics are:

1. Number and Quantity
From preschool through high school, students are continually extending their concept of numbers as they build an understanding of whole numbers, rational numbers, real numbers, and complex numbers. As they engage in real-world mathematical problems, they conceive of quantities, numbers with associated units. Students learn that numbers are governed by properties and understand these properties lead to fluency with operations.

2. Algebra and Functions
Algebraic thinking is about understanding and using numbers, and students' work in this area helps them extend the arithmetic of early grades to expressions, equations, and functions in later grades. This mathematics is applied to real-world problems as students use numbers, expressions, and equations to model the world. The mathematics of this standard is closely related to that of Number and Quantity.

3. Data Analysis, Statistics, and Probability
From the early grades, students gather, display, summarize, examine, and interpret data to discover patterns and deviations from patterns. Measurement is used to generate, represent and analyze data. Working with data and an understanding of the principles of probability lead to a formal study of statistics in middle in high school. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

4. Geometry
Students' study of geometry allows them to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, and engage in logical reasoning. Students learn that geometry is useful in representing, modeling, and solving problems in the real world as well as in mathematics.

Modeling Across the High School Standards

A star symbol (★) in the high school standards represents grade level expectations and evidence outcomes that make up a mathematical modeling standards category.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. (For more on modeling, see Appendix: Modeling Cycle.)

How to Read the Colorado Academic Standards

CONTENT AREA
Grade Level, Standard Category

COLORADO Department of Education

Prepared Graduates:
The *PG Statements* represent concepts and skills that all students who complete the Colorado education system must master to ensure their success in postsecondary and workforce settings.

Grade Level Expectation:
The *GLEs* are an articulation of the concepts and skills for a grade, grade band, or range that students must master to ensure their progress toward becoming a prepared graduate.

Evidence Outcomes

The *EOs* describe the evidence that demonstrates that a student is meeting the GLE at a mastery level.

Academic Context and Connections

The *ACCs* provide context for interpreting, connecting, and applying the content and skills of the GLE. This includes the *Colorado Essential Skills*, which are the critical skills needed to prepare students to successfully enter the workforce or educational opportunities beyond high school embedded within statute (C.R.S. 22-7-1005) and identified by the Colorado Workforce Development Committee.

The ACCs contain information unique to each content area. Content-specific elements of the ACCs are described below.

Colorado Academic Standards Content Area

Grade Level, Standard Category 2020 Colorado Academic Standards GLE Code

Academic Context and Connections in Mathematics:

Colorado Essential Skills and Mathematical Practices: These statements describe how the learning of the content and skills described by the GLE and EOs connects to and supports the development of the *Colorado Essential Skills* and *Standards for Mathematical Practice* named in the parentheses.

Inquiry Questions: The sample question that are intended to promote deeper thinking, reflection, and refined understandings precisely related to the GLE.

Coherence Connections: These statements relate how the GLE relates to content within and across grade levels. The first statement indicates if a GLE is *major*, *supporting*, or *additional* work of the grade. Between 65% and 85% of the work of each grade (with P-2 at the high end of that range) should be focused on the GLEs labeled as **major work**. The remainder of the time should focus on **supporting work** and **additional work**, where it can appropriately support and compliment students' engagement in major work. **Advanced outcomes**, marked with a (+), represent content best saved for upper-level math courses in a student's final three semesters of high school. The remaining statements describe how the GLE and EOs build from content learned in prior grades, connects to content in the same grade, and supports learning in later grades.

Prepared Graduates:

MP6. Attend to precision.

MP7. Look for and make use of structure.

Grade Level Expectation:

3.NBT.A. Number & Operations in Base Ten: Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of algorithms may be used.

Evidence Outcomes

Students Can:

1. Use place value understanding to round whole numbers to the nearest 10 or 100. (CCSS: 3.NBT.A.1)
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS: 3.NBT.A.2)
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations. (CCSS: 3.NBT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly exhibit understanding of a variety of strategies when performing multi-digit arithmetic. (Personal Skills: Adaptability/Flexibility)
2. Demonstrate place value understanding by precisely referring to digits according to their place value. (MP6)
3. Recognize and use place value and properties of operations to structure algorithms and other representations of multi-digit arithmetic. (MP7)

Inquiry Questions:

1. How is rounding whole numbers to the nearest 10 or 100 useful?
2. Do different strategies for solving lead to different answers when we add or subtract? Why or why not?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 2, students use place value understanding and properties of operations to add and subtract fluently within 100.
3. This expectation connects to other ideas in Grade 3: (a) an understanding of multiplication, (b) knowing the relationship between multiplication and division, and (c) the concept of area and its relationship to multiplication and division.
4. In Grade 4, students generalize place value understanding for multi-digit whole numbers and use that understanding and the properties of operations to perform multi-digit arithmetic.

COLORADO Department of Education

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.NF.A. Number & Operations—Fractions: Develop understanding of fractions as numbers.

Evidence Outcomes

Students Can:

1. Describe a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$. (CCSS: 3.NF.A.1)
2. Describe a fraction as a number on the number line; represent fractions on a number line diagram. (CCSS: 3.NF.A.2)
   a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. (CCSS: 3.NF.A.2.a)
   b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line. (CCSS: 3.NF.A.2.b)
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (CCSS: 3.NF.A.3)
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (CCSS: 3.NF.A.3.a)
   b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model. (CCSS: 3.NF.A.3.b)
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express* 3 *in the form* $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and* 1 *at the same point of a number line diagram.* (CCSS: 3.NF.A.3.c)
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. (CCSS: 3.NF.A.3.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly describe fractions both as parts of other numbers but also as numbers themselves. (Personal Skills: Adaptability/Flexibility)
2. Analyze and use information presented visually (for example, number lines, fraction models, and diagrams representing parts and wholes) that support an understanding of fractions as numbers. (Entrepreneurial Skills: Literacy/Reading)
3. Reason about the number line in a new way by understanding and using fractional parts between whole numbers. (MP2)
4. Critique the reasoning of others when comparing fractions that may refer to different wholes. (MP3)
5. Use the structure of fractions to locate and compare fractions on a number line. (MP7)

Inquiry Questions:

1. How does the denominator of a unit fraction connect to the number of unit fractions that must be added to make a whole?
2. When the numerators of two different fractions are the same, how can the denominators be used to compare them?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students (a) relate addition and subtraction to length, (b) measure and estimate lengths in standard units, and (c) reason with shapes and their attributes, including partitioning circles and rectangles into halves, thirds, and fourths.
3. In Grade 3, this expectation connects to the solving of problems involving measurement and estimation of intervals of time, liquid volumes, and mass of objects and is further supported by the expectation to represent and interpret data.
4. In Grade 4, students build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers and extend their understanding of fraction equivalence and ordering. In Grade 6, students apply and extend previous understandings of numbers (including fractions) to the system of rational numbers.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.

Grade Level Expectation:

3.OA.A. Operations & Algebraic Thinking: Represent and solve problems involving multiplication and division.

Evidence Outcomes

Students Can:

1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7. (CCSS: 3.OA.A.1)
2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$. (CCSS: 3.OA.A.2)
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (see Appendix, Table 2) (CCSS: 3.OA.A.3)
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \_ \div 3$, $6 \times 6 = ?$ (CCSS: 3.OA.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving multiples and parts using multiplication and division. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of missing numbers in equations by using the relationship between multiplication and division. (MP1)
3. Reason abstractly about numbers of groups and the size of groups to make meaning of the quantities involved in multiplication and division. (MP2)
4. Use arrays to represent whole-number multiplication and division problems. (MP4)

Inquiry Questions:

1. How can an array be decomposed in a way that connects it to known multiplication facts? How can arrays be used to write and solve multiplication problems?
2. How can the area and one side of a rectangle be used to write and solve a division problem?
3. How could the number of dots in an array be counted without counting them one by one?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. In Grade 3, this expectation connects to understanding properties of multiplication, the relationship between multiplication and division, and to fluently multiplying and dividing within 100.
4. In Grade 4, students (a) use the four operations with whole numbers to solve problems, (b) build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers, and (c) solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. In Grade 5, students apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.OA.B. Operations & Algebraic Thinking: Apply properties of multiplication and the relationship between multiplication and division.

Evidence Outcomes

Students Can:

5. Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If* $6 \times 4 = 24$ *is known, then* $4 \times 6 = 24$ *is also known. (Commutative property of multiplication.)* $3 \times 5 \times 2$ *can be found by* $3 \times 5 = 15$*, then* $15 \times 2 = 30$*, or by* $5 \times 2 = 10$*, then* $3 \times 10 = 30$*. (Associative property of multiplication.) Knowing that* $8 \times 5 = 40$ *and* $8 \times 2 = 16$*, one can find* 8×7 *as* $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$*. (Distributive property.)* (CCSS: 3.OA.B.5)
6. Interpret division as an unknown-factor problem. *For example, find* $32 \div 8$ *by finding the number that makes* 32 *when multiplied by* 8. (CCSS: 3.OA.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Flexibly work with different but related arrangements of factors and products or dividends, divisors, and quotients. (Personal Skills: Adaptability/Flexibility)
2. Use properties of operations to argue for or against the equivalence of different expressions. (MP3)
3. Be specific with explanations and symbols when describing operations using multiplication and division. (MP6)
4. Use the relationship between multiplication and division to rewrite division problems as multiplication. (MP7)

Inquiry Questions:

1. What are all of the equations that can be written to represent the relationship between the area of a (specific) rectangle and its side lengths?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. This expectation connects to other ideas in Grade 3: (a) multiplication and division within 100, (b) solving problems involving the four operations and identifying and explaining patterns in arithmetic, (c) understanding properties of multiplication and the relationship between multiplication and division, and (d) understanding concepts of area and the relationship to multiplication and division.
4. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

3.OA.C. Operations & Algebraic Thinking: Multiply and divide within 100.

Evidence Outcomes

Students Can:

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. (CCSS: 3.OA.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Efficiently solve multiplication and division problems by using facts committed to memory. (Professional Skills: Task/Time Management)
2. Recognize the relationship between skip counting and the solutions to problems involving multiplication and division. (MP7)

Inquiry Questions:

1. How can I use multiplication facts that I know to solve multiplication problems I do not yet know? (for example, using $5 \times 4 + 2 \times 4$ to solve 7×4)?
2. How can I use models and strategies to show what I know about multiplication?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students work with equal groups of objects to gain foundations for multiplication.
3. In Grade 3, this expectation connects with representing and solving problems involving the four operations.
4. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic, solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and gain familiarity with factors and multiples.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP6. Attend to precision.

Grade Level Expectation:

3.OA.D. Operations & Algebraic Thinking: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Evidence Outcomes

Students Can:

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This evidence outcome is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order of operations when there are no parentheses to specify a particular order.) (CCSS: 3.OA.D.8)
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* (CCSS: 3.OA.D.9)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems involving the four operations. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Explain patterns in arithmetic. (MP3)
3. Mathematically model changes in quantities described in real-world contexts using the appropriate numbers, operations, symbols, and letters to represent unknowns. (MP4)
4. Complement arithmetic strategies with mental computation and estimation to assess answers for accuracy. (MP6)

Inquiry Questions:

1. How can a visual model support making sense of and solving word problems?
2. How can the patterns in addition and/or multiplication tables help predict probable solutions to a given problem?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students represent and solve one- and two-step word problems involving addition and subtraction.
3. This expectation connects to several ideas in Grade 3: (a) representing and solving problems involving multiplication and division, (b) multiplying and dividing within 100, (c) solving problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects, and (d) using concepts of area and relating area to multiplication and to addition.
4. In Grade 4, students use the four operations with whole numbers to solve problems.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

3.MD.A. Measurement & Data: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

Evidence Outcomes

Students Can:

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. (CCSS: 3.MD.A.1)
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (This excludes compound units such as cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (This excludes multiplicative comparison problems, such as problems involving notions of "times as much." See Appendix, Table 2.) (CCSS: 3.MD.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use units of measurement appropriate to the type and magnitude of the quantity being measured. (Professional Skills: Information Literacy)
2. Make sense of problems involving measurement by building on real-world knowledge of time and objects and an understanding of the relative sizes of units. (MP1)
3. Represent problems of time and measurement with equations, drawings, or diagrams. (MP4)
4. Use appropriate measures and measurement instruments for the quantities given in a problem. (MP5)

Inquiry Questions:

1. How can elapsed time be modeled on a number line to support the connection to addition and subtraction?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units and work with time and money.
3. In Grade 3, this expectation connects to developing an understanding of fractions as numbers, solving problems involving the four operations, and identifying and explaining patterns in arithmetic.
4. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP4. Model with mathematics.

Grade Level Expectation:

3.MD.B. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent* 5 *pets.* (CCSS: 3.MD.B.3)
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (CCSS: 3.MD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze data to distinguish the factual evidence offered, to reason about judgments, to draw conclusions, and to speculate about ideas the data represents. (Entrepreneurial Skills: Literacy/Reading)
2. Abstract real-world quantities into scaled graphs. (MP2)
3. Model real-world quantities with statistical representations such as bar graphs and line graphs. (MP4)

Inquiry Questions:

1. How can working with pictures and bar graphs connect mathematics to the world around us?
2. How does changing the scale of a bar graph or line plot change the appearance of the data?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 2, students represent and interpret length by measuring objects, make line plots, and use picture and bar graphs to represent categorical data.
3. In Grade 3, this expectation supports developing an understanding of fractions as numbers.
4. In Grade 4, students represent and interpret data by making line plots representing fractional measurements and solving addition and subtraction problems using information presented in line plots.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

3.MD.C. Measurement & Data: Geometric measurement: Use concepts of area and relate area to multiplication and to addition.

Evidence Outcomes

Students Can:

5. Recognize area as an attribute of plane figures and understand concepts of area measurement. (CCSS: 3.MD.C.5)
    a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS: 3.MD.C.5.a)
    b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. (CCSS: 3.MD.C.5.b)
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). (CCSS: 3.MD.C.6)
7. Use concepts of area and relate area to the operations of multiplication and addition. (CCSS: 3.MD.C.7)
    a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (CCSS: 3.MD.C.7.a)
    b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (CCSS: 3.MD.C.7.b)
    c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (CCSS: 3.MD.C.7.c)
    d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. (CCSS: 3.MD.C.7.d)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Defend calculations of area using multiplication and by tiling the area with square units and comparing the results. (MP3)
2. Understand how to use a one-dimensional measurement tool, like a ruler, to make two-dimensional measurements of area. (MP5)
3. Be precise by describing area in square rather than linear units. (MP6)
4. Use areas of rectangles to exhibit the structure of the distributive property. (MP7)

Inquiry Questions:

1. Given three pictures of different rectangles with unknown dimensions, how can you determine which rectangle covers the most area?
2. How does computing the area of a rectangle relate to closed arrays?
3. How can the area of an E-shaped or H-shaped figure be calculated?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units and reason with shapes and their attributes.
3. This expectation connects to other ideas in Grade 3: (a) recognizing perimeter as an attribute of plane figures and distinguishing between linear and area measures, (b) applying properties of multiplication and the relationship between multiplication and division, and (c) solving problems involving the four operations and identifying and explaining patterns in arithmetic.
4. In Grade 4, students solve problems involving measurement and conversion of measurement from a larger unit to a smaller unit. In Grade 5, students relate volume to multiplication and to addition and also extend previous understandings of multiplication and division to multiply and divide fractions.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.

MP4. Model with mathematics.

Grade Level Expectation:

3.MD.D. Measurement & Data: Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Evidence Outcomes

Students Can:

8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS: 3.MD.D.8)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of the relationship between area and perimeter by calculating both for rectangles of varying sizes and dimensions. (MP1)
2. Model perimeters of objects in the world with polygons and the sum of their side lengths. (MP4)

Inquiry Questions:

1. What are all the pairs of side lengths that can create a rectangle with the same area, such as 12 square units?
2. Is it possible for two rectangles to have the same area but different perimeters?
3. Is it possible for two rectangles to have the same perimeter but different areas?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 2, students measure and estimate lengths in standard units.
3. In Grade 3, this expectation connects to understanding concepts of area, relating area to multiplication and to addition, and solving problems involving the four operations.
4. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP3. Construct viable arguments and critique the reasoning of others.

MP7. Look for and make use of structure.

Grade Level Expectation:

3.G.A. Geometry: Reason with shapes and their attributes.

Evidence Outcomes

Students Can:

1. Explain that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (CCSS: 3.G.A.1)
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into* 4 *parts with equal area, and describe the area of each part as* $\frac{1}{4}$ *of the area of the shape.* (CCSS: 3.G.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Work with others to name and categorize shapes. (Civic/Interpersonal Skills: Collaboration/Teamwork)
2. Analyze, compare, and use the properties of geometric shapes to classify them into abstracted categories and describe the similarities and differences between those categories. (MP2)
3. Convince others or critique their reasoning when deciding if a shape belongs to certain categories of polygons. (MP3)
4. Decompose geometric shapes into polygons of equal area. (MP7)

Inquiry Questions:

1. Can you draw a quadrilateral that is not a rhombus, rectangle, or square?
2. (Given two identical squares) Divide each of these squares into four equal parts, but in different ways. If you compare a part of one with a part of the other, are their areas the same? How do you know?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 2, students reason with shapes and their attributes.
3. In Grade 3, this expectation connects to developing an understanding of fractions as numbers.
4. In Grade 4, students draw and identify lines and angles and also classify shapes by properties of their lines and angles.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

4.NBT.A. Number & Operations in Base Ten: Generalize place value understanding for multi-digit whole numbers.

Evidence Outcomes

Students Can:

1. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that* $700 \div 70 = 10$ *by applying concepts of place value and division.* (CCSS: 4.NBT.A.1)
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. (CCSS: 4.NBT.A.2)
3. Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Write multi-digit whole numbers in different forms to support claims and justify reasoning. (Entrepreneurial Skills: Literacy/Writing)
2. Use the structure of the base-ten number system to read, write, compare, and round multi-digit numbers. (MP7)

Inquiry Questions:

1. How do base ten area pieces or representations help with understanding multiplying by 10 or a multiple of 10? How can base ten area pieces be used to represent multiplying by 10 or a multiple of 10?
2. Imagine two four-digit numbers written on paper and some of the digits were smeared. If you saw just 325 ■ and 331 ■, could you determine which number was larger?
3. When is it helpful to use a rounded number instead of the exact number?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students use place value understanding and properties of operations to perform multi-digit arithmetic.
3. In Grade 4, this expectation connects to using the four operations with multi-digit whole numbers to solve measurement and other problems.
4. In Grade 5, students extend their understanding of place value to decimals, and read, write, and compare decimals to thousandths.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.NBT.B. Number & Operations in Base Ten: Use place value understanding and properties of operations to perform multi-digit arithmetic.

Evidence Outcomes

Students Can:

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. (CCSS: 4.NBT.B.4)
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.B.5)
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 4.NBT.B.6)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve multi-digit arithmetic problems. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Explain the process and result of multi-digit arithmetic. (MP3)
3. Precisely and efficiently add and subtract multi-digit numbers. (MP6)
4. Use the structure of place value to support the organization of mental and written multi-digit arithmetic strategies. (MP7)

Inquiry Questions:

1. How can a visual model be used to demonstrate the relationship between multiplication and division?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students use place value understanding and properties of operations to add and subtract within 1000 and to multiply and divide within 100.
3. In Grade 4, this expectation connects to using the four operations with whole numbers to solve problems.
4. In Grade 5, students understand the place value of decimals and perform operations with multi-digit whole numbers and with decimals to hundredths.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.NF.A. Number & Operations—Fractions: Extend understanding of fraction equivalence and ordering.

Evidence Outcomes

Students Can:

1. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (CCSS: 4.NF.A.1)
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. (CCSS: 4.NF.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explain the equivalence of fractions. (MP3)
2. Use visual models and benchmark fractions as tools to aid in fraction comparison. (MP5)
3. Precisely refer to numerators, denominators, parts, and wholes when explaining fraction equivalence and comparing fractions. (MP6)
4. Use 1, the multiplicative identity, to create equivalent fractions by structuring 1 in the fraction form $\frac{n}{n}$. (MP7)

Inquiry Questions:

1. Why does it work to compare fractions either by finding common numerators or by finding common denominators?
2. How can you be sure that multiplying a fraction by $\frac{n}{n}$ does not change the fraction's value?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students develop an understanding of fractions as numbers and the meaning of the denominator of a unit fraction.
3. In Grade 4, this expectation connects to building fractions from unit fractions, using decimal notation and comparing decimal fractions, and using the four operations with whole numbers to solve problems.
4. In Grade 5, students use equivalent fractions as a strategy to add and subtract fractions with unlike denominators and apply and extend previous understandings of multiplication and division to fractions.

Prepared Graduates:

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.NF.B. Number & Operations—Fractions: Build fractions from unit fractions.

Evidence Outcomes

Students Can:

3. Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. (CCSS: 4.NF.B.3)
    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS: 4.NF.B.3.a)
    b. Decompose a fraction into a sum of fractions with like denominators in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$. (CCSS: 4.NF.B.3.b)
    c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS: 4.NF.B.3.c)
    d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (CCSS: 4.NF.B.3.d)
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS: 4.NF.B.4)
    a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as the product* $5 \times \frac{1}{4}$, *recording the conclusion by the equation* $\frac{5}{4} = 5 \times \frac{1}{4}$. (CCSS: 4.NF.B.4.a)
    b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \times \frac{2}{5}$ *as* $6 \times \frac{1}{5}$, *recognizing this product as* $\frac{6}{5}$. *(In general,* $n \times \frac{a}{b} = \frac{n \times a}{b}$.*)* (CCSS: 4.NF.B.4.b)
    c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat* $\frac{3}{8}$ *of a pound of roast beef, and there will be* 5 *people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (CCSS: 4.NF.B.4.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use the structure of fractions to perform operations with fractions and to understand and explain how the operations connect to the structure of fractions. (MP7)
2. Recognize the mathematical connections between the indicated operations with fractions and the corresponding operations with whole numbers. (MP8)

Inquiry Questions:

1. How is the addition of unit fractions similar to counting whole numbers?
2. How does multiplying two whole numbers relate to multiplying a fraction by a whole number?
3. (Given two fractions with like denominators, each of which is less than $\frac{1}{2}$) Before adding these two fractions, can you predict whether the sum will be greater than or less than 1? How do you know?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students develop understanding of fractions as numbers and represent and solve problems involving multiplication and division.
3. This expectation connects to other ideas in Grade 4: (a) using decimal notation for fractions and comparing decimal fractions, (b) using the four operations with whole numbers to solve problems, (c) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit, and (d) representing and interpreting data.
4. In Grade 5, students use equivalent fractions as a strategy to add and subtract fractions with unlike denominators and apply and extend previous understandings of multiplication to decimals.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.NF.C. Number & Operations—Fractions: Use decimal notation for fractions, and compare decimal fractions.

Evidence Outcomes

Students Can:

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) *For example, express* $\frac{3}{10}$ *as* $\frac{30}{100}$, *and add* $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$. (CCSS: 4.NF.C.5)
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite* 0.62 *as* $\frac{62}{100}$; *describe a length as* 0.62 *meters; locate* 0.62 *on a number line diagram.* (CCSS: 4.NF.C.6)
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model. (CCSS: 4.NF.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Approach adding, subtracting, and comparing problems with fractions and decimal fractions by reasoning about their values before or instead of applying an algorithm. (MP1)
2. Draw fraction models to reason about and compute with decimal fractions. (MP5)
3. Make use of the structure of place value to express and compare decimal numbers in tenths and hundredths. (MP7)

Inquiry Questions:

1. How does a fraction with a denominator of 10 or 100 relate to its decimal quantity?
2. How can visual models help to compare two decimal quantities?
3. How is locating a decimal on a number line similar to locating a fraction on a number line?

Coherence Connections:

1. This expectation represents major work of the grade.
2. This expectation connects to several ideas in Grade 4: (a) extending understanding of fraction equivalence and ordering, (b) building fractions from unit fractions, and (c) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
3. In Grade 5, students understand the decimal place value system and use it with the four operations.

Prepared Graduates:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP7. Look for and make use of structure.

Grade Level Expectation:

4.OA.A. Operations & Algebraic Thinking: Use the four operations with whole numbers to solve problems.

Evidence Outcomes

Students Can:

1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS: 4.OA.A.1)
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Appendix, Table 2) (CCSS: 4.OA.A.2)
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make sense of multi-step word problems by understanding the relationships between known and unknown quantities. (MP1)
2. Reason quantitatively with word problems by considering the units involved and how the quantities they describe increase or decrease with addition and subtraction or scale with multiplication and division. (MP2)
3. Use mathematics to model real-world problems requiring operations with whole numbers and contextually interpret remainders when they arise. (MP4)
4. Look for structures of commutativity and inverses of operations in solving whole number problems with the four operations. (MP7)

Inquiry Questions:

1. What makes a multiplicative comparison different from an additive comparison?
2. How can you recognize whether a comparison is multiplicative or additive?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 3, students represent and solve problems involving multiplication and division, apply properties of multiplication and the relationship between multiplication and division, solve problems involving the four operations, and identify and explain patterns in arithmetic.
3. This expectation connects to other ideas in Grade 4: (a) using place value understanding and properties of operations to perform multi-digit arithmetic, (b) extending understanding of fraction equivalence and ordering, (c) building fractions from unit fractions, and (d) solving problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4. In Grade 5, students apply and extend previous understandings of multiplication and division to multiply and divide fractions by fractions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make use of structure.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.OA.B. Operations & Algebraic Thinking: Gain familiarity with factors and multiples.

Evidence Outcomes

Students Can:

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS: 4.OA.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Reason quantitatively to recognize that a number is a multiple of each of its factors. (MP2)
2. Use the relationship between factors and multiples for whole numbers. (MP7)
3. Look for, identify, and explain the regularities in determining whether a given number is a multiple of a given one-digit number and in determining if a given number is prime or composite. (MP8)

Inquiry Questions:

1. How can you use arrays to explore and determine all of the factors of a given number?
2. How are multiples and factors helpful in solving problems related to fractional parts of a whole number, such as $\frac{3}{5}$ of 20?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 3, students multiply and divide within 100.
3. In Grade 6, students compute fluently with multi-digit numbers, find common factors and multiples, and extend previous understandings of arithmetic to algebraic expressions.

Prepared Graduates:

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

4.OA.C. Operations & Algebraic Thinking: Generate and analyze patterns.

Evidence Outcomes

Students Can:

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* (CCSS: 4.OA.C.5)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Explore and generate sequences of numbers or shapes that can be described mathematically. (Entrepreneurial Skills: Creativity/Innovation)
2. Notice when calculations are repeated and describe patterns in generalized, mathematical ways. (MP8)

Inquiry Questions:

1. If you were given a rule to add 4 to a starting number then to each number that follows, can you generate a sequence of odd numbers? How?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 3, students solve problems involving the four operations and identify and explain patterns in arithmetic.
3. In Grade 5, students analyze pairs of patterns created from two given rules and describe and graph the corresponding relationships.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.

Grade Level Expectation:

4.MD.A. Measurement & Data: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Evidence Outcomes

Students Can:

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that* 1 *ft is* 12 *times as long as* 1 *in. Express the length of a* 4 *ft snake as* 48 *in. Generate a conversion table for feet and inches listing the number pairs* (1,12)*,* (2,24)*,* (3,36)*,* … (CCSS: 4.MD.A.1)
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS: 4.MD.A.2)
3. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* (CCSS: 4.MD.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Define quantities in measurement problems with both their magnitude and unit. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make sense of quantities, their units, and their relationships in problem solving situations. (MP2)
3. Model real-world problems involving area and perimeter with equations, diagrams, and formulas, and use them to solve problems. (MP4)
4. Generate and use conversion tables to aid in measurement conversions, and represent measurement quantities on scaled line diagrams. (MP5)

Inquiry Questions:

1. How can you use what you know about place value to convert between km, m and cm? Does this also work for measurement of time (s, m, h)? Why or why not?
2. How can visual models help to make sense of measurement problems and intervals of time?
3. How many liters of juice are needed to fill 35 cups of 225 ml each?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 3, students solve problems involving multiplication and division and solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3. In Grade 4, this expectation connects to building fractions from unit fractions, using decimal notation for fractions, comparing decimal fractions, and using the four operations with whole numbers to solve problems.
4. In Grade 5, students apply and extend previous understandings of multiplication and division, convert like measurement units within a given measurement system, and relate volume to multiplication and to addition.

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Prepared Graduates:

MP5. Use appropriate tools strategically.

Grade Level Expectation:

4.MD.B. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

4. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* (CCSS: 4.MD.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Read and represent measurements recorded on line plots. (Professional Skills: Information Literacy)
2. Use a line plot to represent measurement data and to calculate measurement sums and differences. (MP5)

Inquiry Questions:

1. Why is it helpful to organize data in line plots?
2. When might you see fractions in real-world data?
3. Why is it important to establish the whole when plotting fractions on a line plot?
4. How do labels help the reader determine the size of the numbers represented in a line plot?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 3, students represent and interpret data using picture graphs and scaled bar graphs.
3. In Grade 4, this expectation connects with building fractions from unit fractions.
4. In Grade 5, students represent and interpret data with line plots and solve problems using fractional measurements and all four operations.

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Prepared Graduates:

MP2. Reason abstractly and quantitatively.

Grade Level Expectation:

4.MD.C. Measurement & Data: Geometric measurement: Understand concepts of angle and measure angles.

Evidence Outcomes

Students Can:

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: (CCSS: 4.MD.C.5)
    a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. (CCSS: 4.MD.C.5.a)
    b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. (CCSS: 4.MD.C.5.b)
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (CCSS: 4.MD.C.6)
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (CCSS: 4.MD.C.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze and measure the size of angles in real-world and mathematical problems. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason abstractly and quantitatively about angles and angular measurement. (MP2)

Inquiry Questions:

1. How is measuring angles with a protractor similar to measuring line segments with a ruler?
2. We can describe the fraction $\frac{3}{100}$ as $\frac{1}{100}+\frac{1}{100}+\frac{1}{100}$. How does this apply to the measurement of angles, such as an angle of 3 degrees?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 4, this expectation connects with drawing and identifying lines and angles, classifying shapes by properties of their lines and angles, and with understanding a fraction as a sum of unit fractions.
3. In Grade 7, students solve real-world and mathematical problems involving angle measure, area, surface area, and volume.

Prepared Graduates:

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

Grade Level Expectation:

4.G.A. Geometry: Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Evidence Outcomes

Students Can:

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (CCSS: 4.G.A.1)
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS: 4.G.A.2)
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. (CCSS: 4.G.A.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Make observations and draw conclusions about the classification of two-dimensional figures based on the presence or absence of specified attributes. (Entrepreneurial Skills: Inquiry/Analysis)
2. Use appropriate tools strategically to draw lines (parallel, perpendicular, lines of symmetry), line segments, rays, and angles (right, acute, obtuse). (MP5)
3. Identify ways in which a shape is structured such that it displays line symmetry. (MP7)

Inquiry Questions:

1. Where do you see parallel lines, perpendicular lines, or lines of symmetry in the real world?
2. What kind of angle can you find most often in the real world: right, acute, or obtuse? Why do you think that is the case?
3. What kinds of shapes have many lines of symmetry and what kinds of shapes have no lines of symmetry?
4. In what ways might the lines of symmetry for a shape be related to dividing the shape into fractional parts?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students create composite shapes, recognize and draw shapes having specified attributes, and understand that shapes with shared attributes can define a larger category.
3. In Grade 4, this expectation connects with understanding concepts of angle and measuring angles.
4. In Grade 5, students classify two-dimensional figures into categories based on their properties.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make use of structure.

Grade Level Expectation:

5.NBT.A. Number & Operations in Base Ten: Understand the place value system.

Evidence Outcomes

Students Can:

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. (CCSS: 5.NBT.A.1)
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. (CCSS: 5.NBT.A.2)
3. Read, write, and compare decimals to thousandths. (CCSS: 5.NBT.A.3)
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \frac{1}{10} + 9 \times \frac{1}{100} + 2 \times \frac{1}{1000}$. (CCSS: 5.NBT.A.3.a)
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. (CCSS: 5.NBT.A.3.b)
4. Use place value understanding to round decimals to any place. (CCSS: 5.NBT.A.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Persist in making sense of how fractions can represent decimal place values. (Personal Skills: Perseverance/Resilience)
2. Abstract place value reasoning with whole numbers to decimal numbers. (MP2)
3. See the structure of place value as not just a making of tens with greater place values, but a making of tenths with lesser place values. (MP7)

Inquiry Questions:

1. How can you show visually the relationships between 25, 2.5 and 0.25? How can you show these relationships with equations?
2. Can all decimals be written as fractions? Why or why not?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 4, students generalize place value understanding for multi-digit whole numbers, use decimal notation for fractions, and compare decimal fractions.
3. In Grade 5, this expectation connects with performing operations with multi-digit whole numbers and operations with decimals to hundredths.
4. In Grade 6, students apply and extend previous understandings of arithmetic to algebraic expressions and develop fluency with decimal operations.

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Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

Grade Level Expectation:

5.NBT.B. Number & Operations in Base Ten: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Evidence Outcomes

Students Can:

5. Fluently multiply multi-digit whole numbers using the standard algorithm. (CCSS: 5.NBT.B.5)
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS: 5.NBT.B.6)
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS: 5.NBT.B.7)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Defend calculations with explanations based on properties of operations, equations, drawings, arrays, and other models. (MP3)
2. Use models and drawings to represent and compute with whole numbers and decimals, illustrating an understanding of place value. (MP5)
3. Use the structure of place value to organize computation with whole numbers and decimals. (MP7)

Inquiry Questions:

1. We sometimes use arrays and area models to model multiplication and division of whole numbers. Do these models work for decimal fractions, too? Why or why not?
2. How is computation with decimal fractions similar to and different from computation with whole numbers?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 4, students use place value understanding and properties of operations to perform multi-digit arithmetic.
3. This expectation connects with other ideas in Grade 5: (a) understanding the place value system for decimals, (b) using equivalent fractions as a strategy, (c) applying and extending previous understandings of multiplication and division, and (d) converting like measurement units within a given measurement system.
4. In Grade 6, students compute fluently with multi-digit numbers and find common factors and multiples.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

5.NF.A. Number & Operations—Fractions: Use equivalent fractions as a strategy to add and subtract fractions.

Evidence Outcomes

Students Can:

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. *(In general,* $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$.*)* (CCSS: 5.NF.A.1)
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$, *by observing that* $\frac{3}{7}<\frac{1}{2}$. (CCSS: 5.NF.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Construct viable arguments about the addition and subtraction of fractions with reasoning rooted in the need for like-sized parts. (MP3)
2. Assess the reasonableness of fraction calculations by estimating results using benchmark fractions and number sense. (MP6)
3. Look for structure in the multiplicative relationship between unlike denominators when creating equivalent fractions. (MP7)

Inquiry Questions:

1. It is useful to round decimals when estimating sums and differences of decimal numbers. What would “rounding fractions” look like when estimating sums and differences of fractions?
2. Why don’t we add or subtract the denominators when we are working with fractions?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In Grade 4, students add and subtract fractions and mixed numbers with like denominators, recognize and generate equivalent fractions, and compare fractions with different numerators and denominators.
3. In Grade 5, this expectation connects with multi-digit whole number operations, operations with decimals to hundredths, and representing and interpreting data.
4. In Grade 6, students reason about and solve one-variable equations and inequalities, and in Grade 7, apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Prepared Graduates:

MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.

Grade Level Expectation:

5.NF.B. Number & Operations—Fractions: Apply and extend previous understandings of multiplication and division.

Evidence Outcomes

Students Can:

3. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (CCSS: 5.NF.B.3)
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (CCSS: 5.NF.B.4)
   a. Interpret the product $\frac{a}{b} \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.)* (CCSS: 5.NF.B.4.a)
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS: 5.NF.B.4.b)
5. Interpret multiplication as scaling (resizing), by: (CCSS: 5.NF.B.5)
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. (CCSS: 5.NF.B.5.a)
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1. (CCSS: 5.NF.B.5.b)
6. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (CCSS: 5.NF.B.6)
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) (CCSS: 5.NF.B.7)
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $\frac{1}{3} \div 4$, and use a visual fraction model to show the quotient. Use the*

relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ *because* $\frac{1}{12} \times 4 = \frac{1}{3}$. (CCSS: 5.NF.B.7.a)

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div \frac{1}{5}$, *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div \frac{1}{5} = 20$ *because* $20 \times \frac{1}{5} = 4$. (CCSS: 5.NF.B.7.b)

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if* 3 *people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$*-cup servings are in* 2 *cups of raisins?* (CCSS: 5.NF.B.7.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve problems requiring calculations that scale whole numbers and fractions. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Use fraction models and arrays to interpret and explain fraction calculations. (MP5)
3. Attend carefully to the underlying unit quantities when solving problems involving multiplication and division of fractions. (MP6)
4. Contrast previous understandings of multiplication modeled as equal groups to multiplication as scaling, which is necessary to understand multiplying a fraction or whole number by a fraction, and how the operation of multiplication does not always result in a product larger than both factors. (MP7)

Inquiry Questions:

1. How can you rewrite the fraction $\frac{5}{3}$ with an addition equation? How can you rewrite it with a multiplication equation? How does it make sense that both equations are accurate?
2. If we can describe the product of 5×3 as "three times as big as 5," what does that tell us about the product of $5 \times \frac{1}{2}$? What about $\frac{1}{5} \times \frac{1}{2}$?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students base understanding of multiplication on its connection to addition, groups of equivalent objects, and area models. In Grade 4, students add and subtract fractions and mixed numbers with like denominators, recognize and generate equivalent fractions, and compare fractions with different numerators and denominators.
3. This expectation connects with several others in Grade 5: (a) performing operations with multi-digit whole numbers and with decimals to hundredths, (b) writing and interpreting numerical expressions, and (c) representing and interpreting data.
4. In Grade 6, students (a) understand ratio concepts and use ratio reasoning to solve problems, (b) apply and extend previous understandings of multiplication and division to divide fractions by fractions, (c) reason about and solve one-variable equations and inequalities, and (d) solve real-world and mathematical problems involving area, surface area, and volume.

Prepared Graduates:

MP7. Look for and make use of structure.

Grade Level Expectation:

5.OA.A. Operations & Algebraic Thinking: Write and interpret numerical expressions.

Evidence Outcomes

Students Can:

1. Use grouping symbols (parentheses, brackets, or braces) in numerical expressions, and evaluate expressions with these symbols. (CCSS: 5.OA.A.1)
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add* 8 *and* 7*, then multiply by* 2*" as* $2 \times (8 + 7)$*. Recognize that* $3 \times (18932 + 921)$ *is three times as large as* $18932 + 921$*, without having to calculate the indicated sum or product.* (CCSS: 5.OA.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Write expressions that represent mathematical relationships between quantities. (Entrepreneurial Skills: Literacy/Writing)
2. Look for structures and notation that make the order of operations clear when reading and writing mathematical expressions. (MP7)

Inquiry Questions:

1. How can you describe the relationship between the value of $5 \times (24562 + 951)$ and $24562 + 951$ without making any calculations?
2. Suppose we use the letter a to represent a number. Can you determine the relationship between $4 \times a$ and a without knowing the specific number a represents?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 5, this expectation connects with applying and extending previous understandings of multiplication and division.
3. In Grade 6, students compute fluently with multi-digit numbers, find common factors and multiples, and apply and extend previous understandings of arithmetic to algebraic expressions. In middle and high school, students develop fluency with algebraic expressions as mathematical objects that can be used in more complex mathematical operations.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP8. Look for and express regularity in repeated reasoning.

Grade Level Expectation:

5.OA.B. Operations & Algebraic Thinking: Analyze patterns and relationships.

Evidence Outcomes

Students Can:

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* (CCSS: 5.OA.B.3)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Analyze and compare patterns. (Entrepreneurial Skills: Inquiry/Analysis)
2. Reason quantitatively with patterns by relating sequences of numbers with the rule that generated them. (MP3)
3. Look for repeated reasoning both within individual patterns and in mathematical relationships between pairs of patterns. (MP8)

Inquiry Questions:

1. When you graph the corresponding terms formed by two numerical rules, how are the rules reflected in the graph?
2. How does the relationship between two patterns generated by rules relate to the rules themselves?

Coherence Connections:

1. This expectation is in addition to major work of the grade.
2. In Grade 4, students generate and analyze number or shape patterns and generalize about them.
3. In Grade 6, students understand ratio concepts, use ratio reasoning to solve problems, extend previous understandings of arithmetic to algebraic expressions, and represent and analyze quantitative relationships between dependent and independent variables.

Prepared Graduates:

MP6. Attend to precision.

Grade Level Expectation:

5.MD.A. Measurement & Data: Convert like measurement units within a given measurement system.

Evidence Outcomes

Students Can:

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems. (CCSS: 5.MD.A.1)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Convert measurements to solve real-world problems. (Professional Skills: Information Literacy)
2. Use appropriate precision when converting measurements based on a problem's context. (MP6)

Inquiry Questions:

1. What is happening mathematically when we convert from centimeters to meters? What about when we convert from meters to centimeters?
2. How can you use fractions to change 53 kilograms to grams? How can you use decimals to do this conversion?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 4, students solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
3. In Grade 5, this expectation connects with performing operations with multi-digit whole numbers and with decimals to hundredths.

Prepared Graduates:

MP5. Use appropriate tools strategically.

Grade Level Expectation:

5.MD.B. Measurement & Data: Represent and interpret data.

Evidence Outcomes

Students Can:

2. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* (CCSS: 5.MD.B.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Display fractional measurement data in line plots. (Professional Skills: Information Literacy)
2. Participate in discussions of measurement data using information presented in line plots. (Civic/Interpersonal Skills: Literacy/Oral Expression and Listening)
3. Strategically determine the scale of line plots to represent fractional measurements. (MP5)

Inquiry Questions:

1. (Given a data set of fractional measurements with unlike denominators) What will you consider in deciding how to label the tick marks on the line for your line plot?

Coherence Connections:

1. This expectation supports the major work of the grade.
2. In Grade 4, students represent and interpret data.
3. In Grade 5, this expectation connects with using equivalent fractions and applying and extending previous understandings of multiplication and division.
4. In Grade 6, students develop understanding of statistical variability and summarize and describe distributions.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.
MP5. Use appropriate tools strategically.
MP7. Look for and make use of structure.

Grade Level Expectation:

5.MD.C. Measurement & Data: Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

Evidence Outcomes

Students Can:

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. (CCSS: 5.MD.C.3)
   a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume. (CCSS: 5.MD.C.3.a)
   b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. (CCSS: 5.MD.C.3.b)
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (CCSS: 5.MD.C.4)
5. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. (CCSS: 5.MD.C.5)
   a. Model the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (CCSS: 5.MD.C.5.a)
   b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems. (CCSS: 5.MD.C.5.b)
   c. Use the additive nature of volume to find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems. (CCSS: 5.MD.C.5.c)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Solve real-world problems involving volume. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Make connections between the values being multiplied in a volume formula, the concept of cubic units, and the context within which volume is being calculated. (MP2)
3. Use unit cubes as a tool for finding or estimating volume and compare those results with those obtained with formulas. (MP5)
4. Extend the structure of two-dimensional space and the relationship between arrays and area to three-dimensional space and the relationship between layers of cubes and volume. (MP7)

Inquiry Questions:

1. How are volume and area related in a solid figure?
2. Why is multiplication used when computing the volume of a solid figure, instead of another operation?

Coherence Connections:

1. This expectation represents major work of the grade.
2. In previous grades, students connect area to the operation of multiplication and understand how to represent area problems as multiplication equations.
3. In Grade 6, students solve real-world and mathematical problems involving area of right rectangular prisms with fractional side lengths, using fractional cubic units.

Prepared Graduates:

MP2. Reason abstractly and quantitatively.

MP5. Use appropriate tools strategically.

Grade Level Expectation:

5.G.A. Geometry: Graph points on the coordinate plane to solve real-world and mathematical problems.

Evidence Outcomes

Students Can:

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). (CCSS: 5.G.A.1)
2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (CCSS: 5.G.A.2)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Use the first quadrant of the coordinate plane to represent real-world and mathematical problems. (Entrepreneurial Skills: Critical Thinking/Problem Solving)
2. Analyze and use information presented visually in a coordinate plane. (Entrepreneurial Skills: Literacy/Reading)
3. Reason quantitatively about a problem by abstracting and representing the situation in the first quadrant of the coordinate plane. (MP2)
4. Use the first quadrant of the coordinate plane as a tool to represent, analyze, and solve problems. (MP5)

Inquiry Questions:

1. What are things in the real world that are designed like a coordinate plane or that use a coordinate system?
2. Why are the axes of the coordinate plane made to form right angles instead of acute and obtuse angles?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In previous grades, students use number lines to represent whole and fractional number distances from zero.
3. In Grade 6, students extend the number line and the coordinate plane to include negative numbers and solve real-world and mathematical problems by graphing points in all four quadrants.

Prepared Graduates:

MP3. Construct viable arguments and critique the reasoning of others.

MP5. Use appropriate tools strategically.

MP7. Look for and make use of structure.

Grade Level Expectation:

5.G.B. Geometry: Classify two-dimensional figures into categories based on their properties.

Evidence Outcomes

Students Can:

3. Explain that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* (CCSS: 5.G.B.3)
4. Classify two-dimensional figures in a hierarchy based on properties. (CCSS: 5.G.B.4)

Academic Context and Connections

Colorado Essential Skills and Mathematical Practices:

1. Observe and analyze attributes of two-dimensional figures to classify them. (Entrepreneurial Skills: Inquiry/Analysis)
2. Critique the reasoning of others' classifications of two-dimensional shapes. (MP3)
3. Strategically use measurement tools to help improve the classification of shapes. (MP5)
4. Look for and use attributes of two-dimensional shapes to classify the shapes in a hierarchy of figures. (MP7)

Inquiry Questions:

1. How can you use the words "always," "sometimes," and "never" to develop a classification of two-dimensional figures?

Coherence Connections:

1. This expectation is in addition to the major work of the grade.
2. In Grade 4, students draw and identify lines and angles and classify shapes by properties of their lines and angles.

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

	Unknown Product	**Group Size Unknown** ("How many in each group? Division)	**Number of Groups Unknown** ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red had costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long as the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first.
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[5] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Table 3. The properties of operations. Here, a, b, and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every a there exists $-a$ so that $a+(-a)=(-a)+a=0$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Table 4. The properties of equality. Here, a, b, and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a=b$, then $b=a$.
Transitive property of equality	If $a=b$ and $b=c$, then $a=c$.
Addition property of equality	If $a=b$, then $a+c=b+c$.
Subtraction property of equality	If $a=b$, then $a-c=b-c$.
Multiplication property of equality	If $a=b$, then $a\times c=b\times c$.
Division property of equality	If $a=b$ and $c\neq 0$, then $a\div c=b\div c$.
,Substitution property of equality	If $a=b$, then b may be substituted for a in any expression containing a.

Table 5. The properties of inequality. Here, a, b, and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a<b$, $a=b$, $a>b$.

If $a>b$ and $b>c$ then $a>c$.

If $a>b$, then $b<a$.

If $a>b$, then $-a<-b$.

If $a>b$, then $a\pm c>b\pm c$.

If $a>b$ and $c>0$, then $a\times c>b\times c$.

If $a>b$ and $c<0$, then $a\times c<b\times c$.

If $a>b$ and $c>0$, then $a\div c>b\div c$.

If $a>b$ and $c<0$, then $a\div c<b\div c$.

Modeling links classroom mathematics and statistics to everyday life, work, and decision making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Some examples of situations requiring modeling might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and financial investments.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.